Predictive nonlinear biplots: maps and trajectories

Karen Vines
Department of Mathematics and Statistics
The Open University
Nonlinear biplots

• Simultaneous representation of observations and variables

• Dissimilarities between observations calculated using an Euclidean embeddable distance function.

• e.g. Clark’s distance, square root of Canberra distance

\[ d^2 = \sum_{k=1}^{p} \left( \frac{x_{ik} - x_{jk}}{x_{ik} + x_{jk}} \right)^2 \]

\[ d^2 = \sum_{k=1}^{p} \left| \frac{x_{ik} - x_{jk}}{x_{ik} + x_{jk}} \right| \]

• Position of points given by classical scaling
Aircraft data

Data originally from Stanley and Miller (1979)

- 21 fighter aircraft
- 4 variables:
  - SPR (specific power)
  - RGF (flight range factor)
  - PLF (payload, as fraction of gross weight)
  - SLF (sustained load factor)
Plot of the aircraft data
Adding variable information

Take approach in Gower and Harding (1988)

• Can calculate position of a new observation \((\mu_1, 0, 0, 0)\).
  – gives position of the value \(\mu_1\) for the first variable
  – joining a series of these points gives an axis for the first variable (SPR)

• But axis lives in high-dimensional space

• Interpolation or prediction matters. Focus here on prediction.
Prediction in biplots

For a given point, \( \alpha \), on the biplot, find the value of \( \mu \) which corresponds where the axis is closest.

Use least squares...

... equivalent to finding \( \mu \) which minimises

\[
\sum_{i=1}^{n} w_i d^2(x_i, \mu)
\]
Prediction map - SPR

Legend

Distance measure: Clark
Axes type: none
Shape parameter 1
Inter-point distances interpretable

Prediction map: SPR
Prediction map - PLF

Legend

Distance measure: Clark
Axes type: none
Shape parameter 1
Inter-point distances interpretable

Prediction map: PLF

Distance Scale

0.001
0.01
0.1
0.2

Color Scale

0.01
0.1
0.2
Normal predictive trajectories

Predictive biplot

Legend

Distance measure: Clark
Axes type: normal
Shape parameter 1
Inter-point distances interpretable

SPR
RGF
PLF
SLF
Prediction map - SPR

Legend
Distance measure: Canberra
Axes type: none
Shape parameter 1
Inter-point distances interpretable

Prediction map: SPR
Adding an arc
Adding an arc
Adding an arc
Adding an arc
Adding an arc
Map and trajectory

Legend

Distance measure: Canberra
Axes type: normal
Shape parameter 1
Inter-point distances interpretable

Prediction map: SPR

Map and trajectory
Distance measure: Canberra
Axes type: normal
Shape parameter 1
Inter-point distances interpretable
Prediction map: PLF
Trajectories only

Legend

Distance measure: Canberra
Axes type: normal
Shape parameter 1
Inter-point distances interpretable

SPR
RGF
PLF
SLF
Performance of trajectories

- Aircraft ‘f’

<table>
<thead>
<tr>
<th>Variable</th>
<th>Trajectory</th>
<th>Map</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPR</td>
<td>1.605</td>
<td>1.605</td>
<td>1.294</td>
</tr>
<tr>
<td>RGF</td>
<td>3.75</td>
<td>3.75</td>
<td>3.75</td>
</tr>
<tr>
<td>PLF</td>
<td>0.166</td>
<td>0.166</td>
<td>0.150</td>
</tr>
<tr>
<td>SLF</td>
<td>1.00</td>
<td>1.00</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Further work

- Decision rule for when two or more predicted values are indicated.
- Calculation of prediction discontinuities.
- Investigation of when non data values are predicted for non-smooth distance functions.