

“THANK YOU FOR BEING  
HERE”

[http://www.ted.com/talks/john\\_francis\\_walks\\_the\\_earth.html](http://www.ted.com/talks/john_francis_walks_the_earth.html)

# Welcome to the 6<sup>th</sup> CARME conference



# The applied mathematics department



- Jérôme PAGES : Professeur, Directeur du laboratoire
- Marine CADORET : Maître de conférences contractuelle
- David CAUSEUR : Professeur
- Thibaut DUTRION : Ingénieur d'étude
- Magalie HOUEE : Ingénieur d'étude
- François HUSSON : Maître de conférences
- Julie JOSSE : Maître de Conférences
- Sébastien LÊ : Maître de conférences
- Marie VERBANCK : Doctorante
- Elisabeth LENAULD, Karine BAGORY : Secrétaires

# TODAY'S TUTORIALS

The tutorials of the day

# Tutorials

- Lê Sébastien: *From one to multiple data tables with FactoMineR*
- Dray Stéphane: *Multivariate analysis of ecological data with ade4*
- Mair Patrick, de Leeuw Jan: *Multidimensional scaling using majorization with smacof*
- Nenadic Oleg, Greenacre Michael: *Correspondence analysis with ca*

What's is common to all those presentations?



*The New York Times*

I READ THE NEWS TODAY  
OH BOY...

The Beatles  
A day in the life

# Data Analysts Captivated by R's Power

*January 6, 2009 from the New York Times*

- “The popularity of R at universities could threaten SAS Institute (...)”

# R You Ready for R?

*January 8, 2009 from the New York Times*

- “Intel Capital has placed the number of R users at 1 million, and Revolution kicks the estimate all the way up to 2 million.”

# What is R?

- R is a **free** software environment for statistical computing and graphics. (<http://www.r-project.org/>)
- R is a **freely** available language and environment for statistical computing and graphics which provides a wide variety of statistical and graphical techniques: linear and nonlinear modelling, statistical tests, time series analysis, classification, clustering, etc. (<http://cran.r-project.org/>)
- R was designed by Ross Ihaka and Robert Gentleman

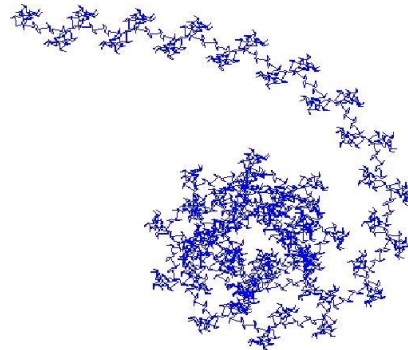
# How R became a must in a decade?

- Its economic model: it's free.
- Mr. Ihaka said: “We could have chosen to be commercial, and we would have sold five copies of the software.”



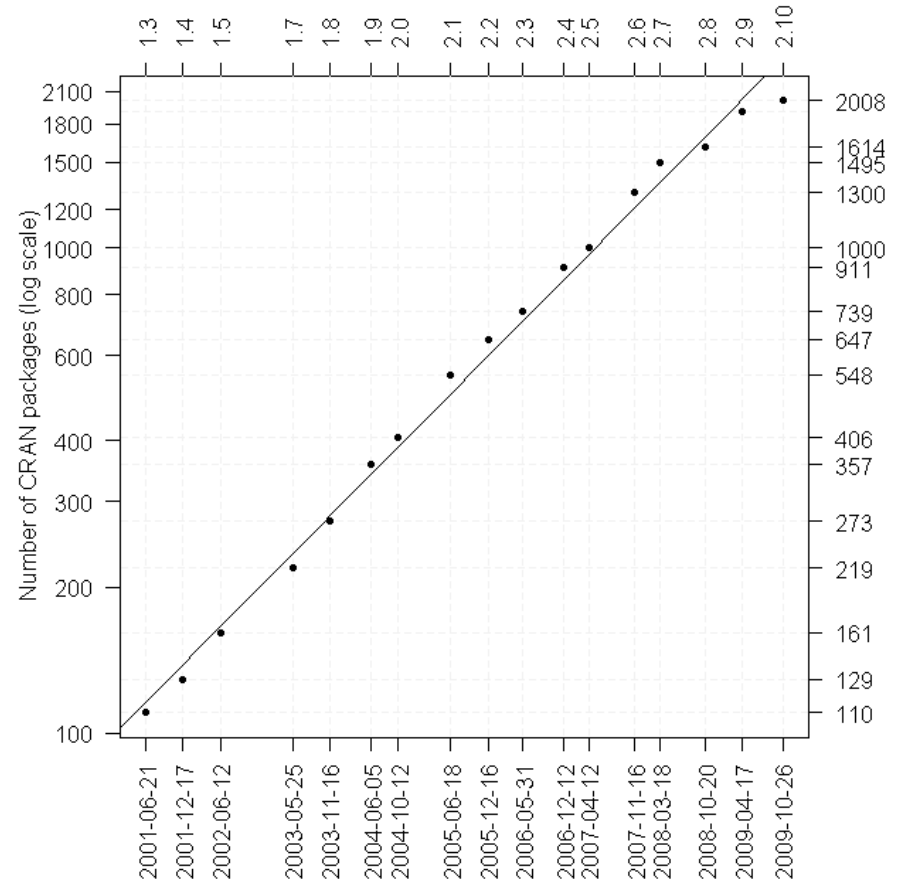
# How R became a must in a decade?

- **The snowball effect:** a figurative term for a process that starts from an initial state of small significance and builds upon itself, becoming larger (graver, more serious), and perhaps potentially dangerous or disastrous (a vicious circle, a "spiral of decline"), though it might be beneficial instead (a virtuous circle) (wikipedia)



# The snowball effect

Version	Packages	Date
<b>1.3</b>	110	21/06/2001
<b>1.4</b>	129	17/12/2001
<b>1.5</b>	161	12/06/2002
<b>1.6</b>	163	
<b>1.7</b>	219	25/05/2003
<b>1.8</b>	273	16/11/2003
<b>1.9</b>	357	05/06/2004
<b>2.0</b>	406	12/10/2004
<b>2.1</b>	548	18/06/2005
<b>2.2</b>	647	16/12/2005
<b>2.3</b>	739	31/05/2006
<b>2.4</b>	911	12/12/2006
<b>2.5</b>	1000	12/04/2007
<b>2.6</b>	1300	16/11/2007
<b>2.7</b>	1495	18/03/2008
<b>2.8</b>	1614	20/10/2008
<b>2.9</b>	1907	17/04/2009
<b>2.10</b>	2008	26/10/2009



# Why create an R package?

(P. Rossi)

There are three good reasons:

- Creating an R package forces you to document your code and provide test examples to insure that it actually works. It will also be much easier to use your code as documentation will only be a ? command away and all of your functions and shared libraries will be available for use.
- If your goal is disseminate your research, this is an ideal way of making sure others have access to your work. It will also increase the probability that eventually your work will be correct. You will also learn more about the properties of your research ideas through the experience of others.
- Ease your sense of guilt by giving back something to this amazing community of volunteers!

# The R packages



# The R packages

- There's an old bulgarian proverb that says « всеки проблем има R решение »
- In other words « each problem has its R package»



# The « sudoku » package

Sudoku Puzzle Generator and Solver



Documentation for package 'sudoku' version 2.2

- [DESCRIPTION file](#).

## Help Pages

[fetchSudokuUK](#)

[generateSudoku](#)

[hintSudoku](#)

[playSudoku](#)

[printSudoku](#)

[readSudoku](#)

[solveSudoku](#)

[writeSudoku](#)

Fetch the daily sudoku puzzle from <http://www.sudoku.org.uk/>

Randomly Generate a Sudoku Puzzle Grid

Give a Hint for a Sudoku Cell

Interactively play a game of Sudoku

Print a Sudoku Grid to the Terminal.

Read a File Containing a Sudoku Grid

Solve a Sudoku Puzzle

Write a Sudoku Grid to a File

# The « FactoMineR » package



<http://factominer.free.fr>

*Journal of statistical software* **FactoMineR: an R package for multivariate analysis**

*FROM MULTIVARIATE TO  
MULTIPLE TABLES DATA  
ANALYSIS...AN OVERVIEW*

Sébastien Lê and Jérôme Pagès

# Objectives

- To understand what can be expected from multiway data methods
- To understand the motivations and the framework of Canonical Analysis (CA)
- To understand Generalized Canonical Analysis (GCA)
- To be able to place Multiple Factor Analysis vs. GCA

# Outline



- What you know
- What you want to do and why you want to do it
- How to do it
- How to do it with R

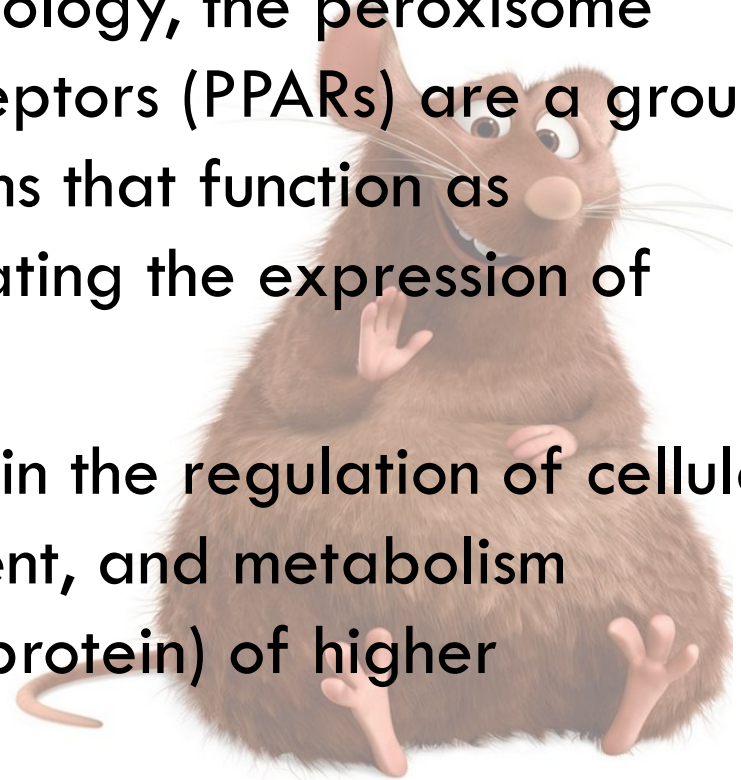
# The data



- 40 mice
- 2 genotypes (wild, PPAR $\alpha$ -deficient)
- 5 diets (dha, efad, lin, ref, tsol)
- 120 genes (expression)
- 21 hepatic fatty acid concentration
- Thanks to Sandrine Lagarrigue, Genetics department-INRA Rennes, for the availability of the data

# PPAR $\alpha$

- In the field of molecular biology, the peroxisome proliferator-activated receptors (PPARs) are a group of nuclear receptor proteins that function as transcription factors regulating the expression of genes.
- PPARs play essential roles in the regulation of cellular differentiation, development, and metabolism (carbohydrate, lipid, and protein) of higher organisms.
- PPAR $\alpha$  are expressed in liver, kidney, heart, muscle, adipose tissue, and others



# The diets

- dha: diet rich in fatty acids of the Omega 3 family and particularly docosahexaenoic acid (DHA), based on fish oil;
- efad (Essential Fatty Acid Deficient): diet based on of saturated fatty acids only, made from hydrogenated coconut oil;
- lin: diet rich in Omega 3, made from linseed oil;
- ref: regime whose contribution in Omega 6 and Omega 3 is adapted for the French population, seven times more Omega 6 than Omega 3;
- tsol: diet rich in Omega 6, based on sunflower oil.

# Issues

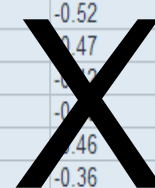


# MY FIRST PCA

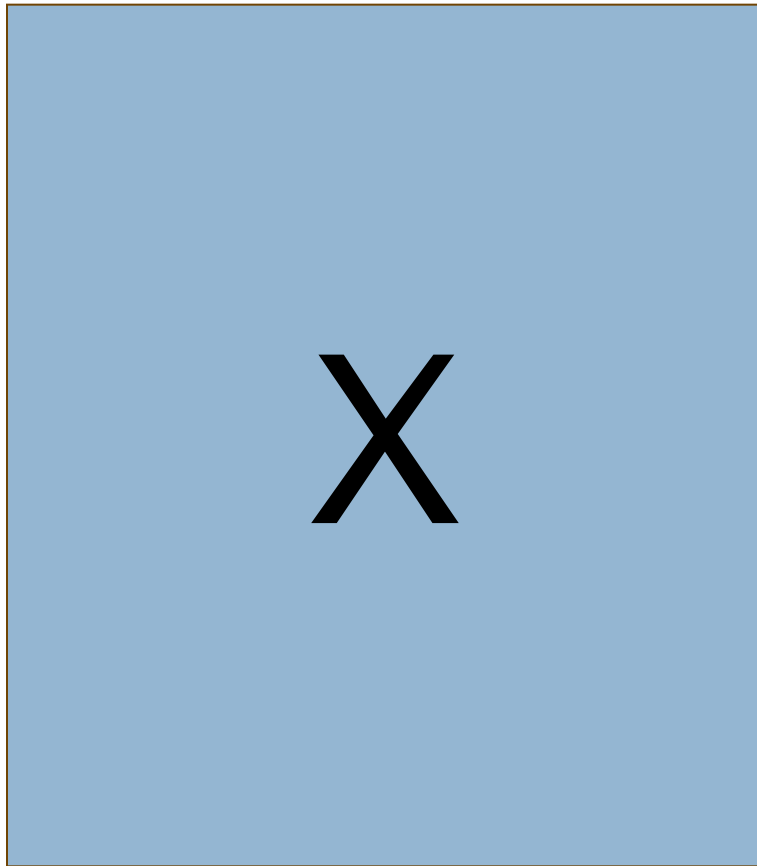
with supplementary qualitative variables (and  
FactoMineR)

# The dataset

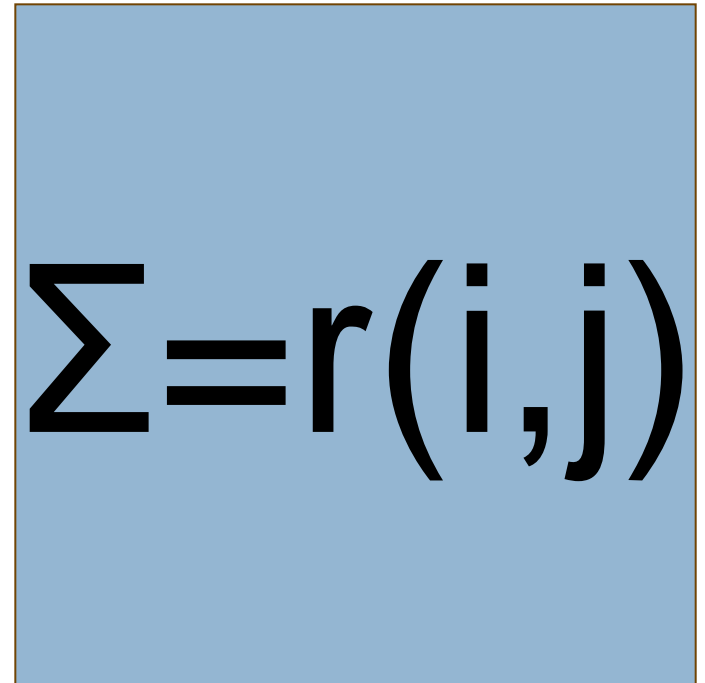
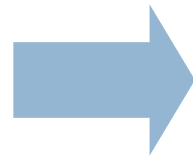
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
1	Souris	Regime	Genotype	C14.0	C16.0	C18.0	C16.1n.9	C16.1n.7	C18.1n.9	X36b4	ACAT1	ACAT2	ACBP	ACC1	ACC2	ACO
2	67	lin	1	0.34	26.45	10.22	0.35	3.10	16.98	-0.42	-0.65	-0.84	-0.34	-1.29	-1.13	-0.93
3	68	toumesol	1	0.38	24.04	9.93	0.55	2.54	20.07	-0.44	-0.68	-0.91	-0.32	-1.23	-1.06	-0.99
4	69	toumesol	1	0.36	23.70	8.96	0.55	2.65	22.89	-0.48	-0.74	-1.1	-0.46	-1.3	-1.09	-1.06
5	75	dha	1	0.22	25.48	8.14	0.49	2.82	21.92	-0.45	-0.69	-0.65	-0.41	-1.26	-1.09	-0.93
6	88	ref	1	0.37	24.80	9.63	0.46	2.85	21.38	-0.42	-0.71	-0.54	-0.38	-1.21	-0.89	
7	89	efad	1	1.70	26.04	6.59	0.66	7.26	28.23	-0.43	-0.69	-0.8	-0.32	-1.13	-0.79	-0.93
8	90	lin	1	0.35	25.94	9.68	0.36	3.60	17.62	-0.53	-0.62	-1	-0.44	-1.22	-1	-0.94
9	92	lin	1	0.34	28.63	9.95	0.29	3.27	17.02	-0.49	-0.69	-0.91	-0.37	-1.29	-1.06	-1.05
10	96	dha	1	0.22	25.34	8.81	0.44	2.36	18.39	-0.36	-0.66	-0.74	-0.39	-1.15	-1.08	-0.88
11	98	efad	1	1.38	28.49	5.63	0.90	7.01	36.68	-0.5	-0.62	-0.79	-0.36	-1.21	-0.82	-0.92
12	99	dha	1	0.26	25.73	8.30	0.43	2.74	21.75	-0.4	-0.6	-0.55	-0.25	-1.22	-1.13	-0.81
13	101	ref	1	0.44	24.28	8.63	0.53	3.33	23.86	-0.52	-0.66	-0.66	-0.41	-1.28	-1.1	-0.95
14	102	toumesol	1	0.32	24.63	9.99	0.45	2.39	17.93	-0.52	-0.63	-0.99	-0.43	-1.24	-0.96	-0.96
15	111	ref	1	0.34	26.04	9.81	0.35	2.36	20.14	-0.47	-0.71	-0.44	-0.45	-1.44	-1.17	-1.02
16	118	toumesol	1	0.35	24.76	9.38	0.54	2.47	19.66	-0.5	-0.66	-0.88	-0.33	-1.24	-1.03	-0.92
17	120	lin	1	0.24	26.46	10.97	0.31	2.81	14.69	-0.4	-0.62	-0.85	-0.42	-1.33	-1.19	-0.91
18	121	efad	1	1.21	23.45	5.59	0.67	6.31	33.84	-0.46	-0.69	-0.45	-0.32	-1.31	-0.93	-1.06
19	123	dha	1	0.30	29.72	8.95	0.45	2.86	17.79	-0.36	-0.58	-0.71	-0.34	-1.2	-0.98	-0.92
20	130	efad	1	1.30	27.00	5.72	0.81	7.86	33.50	-0.35	-0.62	-0.56	-0.24	-1.24	-0.88	-0.95
21	135	ref	1	0.38	24.09	8.22	0.60	3.89	24.61	-0.44	-0.75	-0.7	-0.33	-1.35	-1.04	-0.99
22	281	efad	2	3.24	23.59	2.68	1.11	13.09	35.61	-0.48	-0.71	-0.63	-0.55	-1.27	-1.03	-0.88
23	285	ref	2	0.60	19.95	3.18	1.21	4.89	35.91	-0.5	-0.69	-0.86	-0.58	-1.31	-1.06	-0.88
24	290	toumesol	2	0.38	17.64	6.99	0.74	2.58	21.23	-0.54	-0.69	-1.02	-0.51	-1.32	-1.2	-0.96
25	294	dha	2	0.44	22.73	4.71	0.75	2.27	25.10	-0.39	-0.7	-0.83	-0.47	-1.27	-1.09	-0.91
26	295	toumesol	2	0.47	14.65	4.29	0.66	2.88	23.15	-0.54	-0.63	-0.86	-0.56	-1.35	-1.17	-0.88
27	303	ref	2	0.64	20.49	2.71	1.09	4.05	38.32	-0.35	-0.52	-0.74	-0.4	-1.07	-0.83	-0.73
28	305	ref	2	0.52	18.44	5.21	0.87	3.37	32.04	-0.46	-0.69	-0.53	-0.51	-1.33	-0.99	-0.9
29	307	lin	2	0.49	17.72	6.02	0.58	3.77	21.04	-0.42	-0.56	-0.86	-0.57	-1.23	-1.04	-0.75
30	319	dha	2	0.40	21.70	6.12	0.78	2.07	22.49	-0.51	-0.7	-0.68	-0.46	-1.34	-1.13	-0.94
31	321	lin	2	0.61	16.25	4.55	0.61	5.42	21.43	-0.47	-0.74	-0.88	-0.58	-1.3	-1.03	-0.83
32	325	efad	2	3.19	22.91	3.60	0.99	13.90	33.55	-0.54	-0.64	-0.39	-0.46	-1.3	-1.03	-0.85



# The dataset

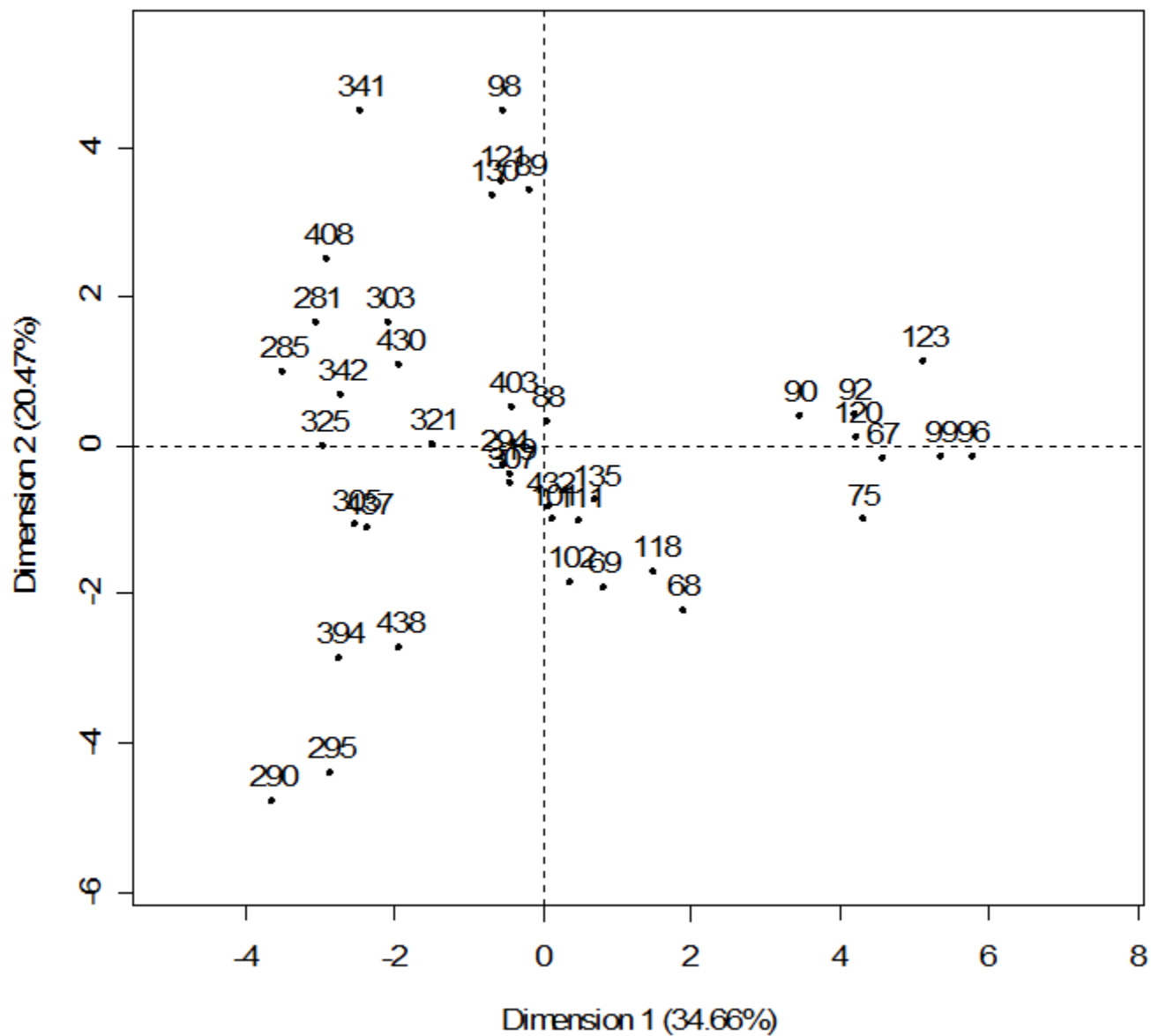


$M(n,p)$

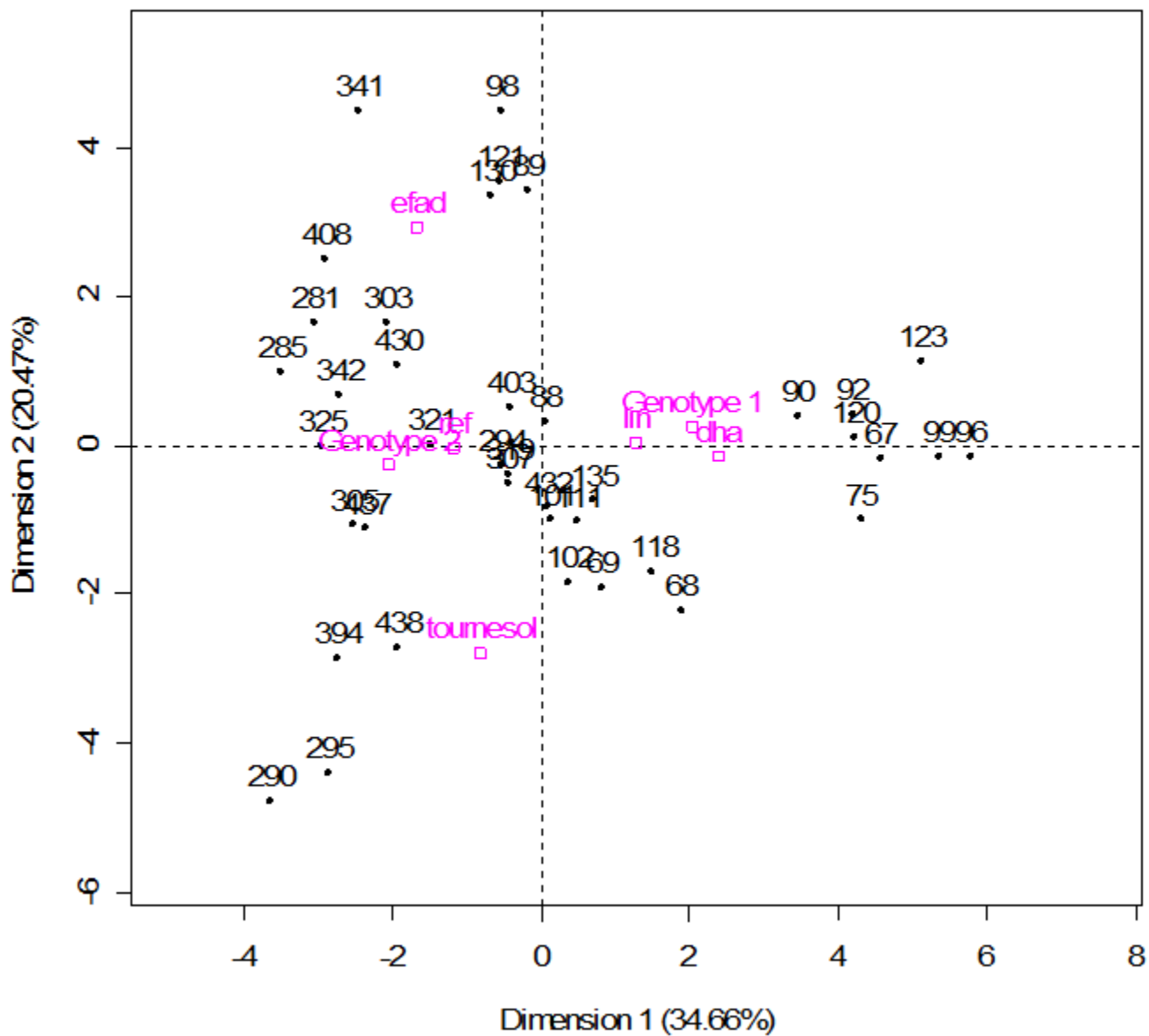


$M(p,p)$

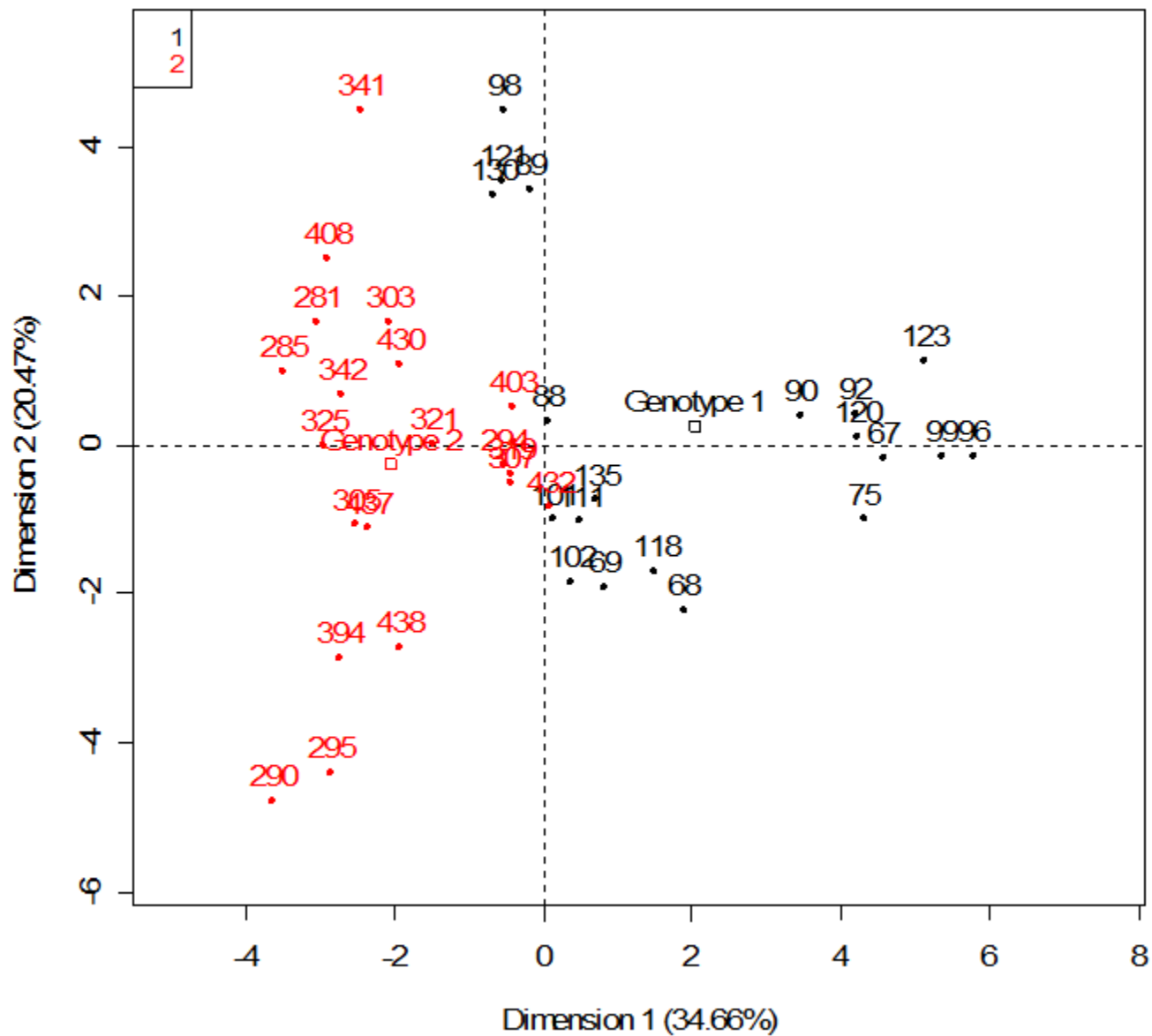
Individuals factor map (PCA)



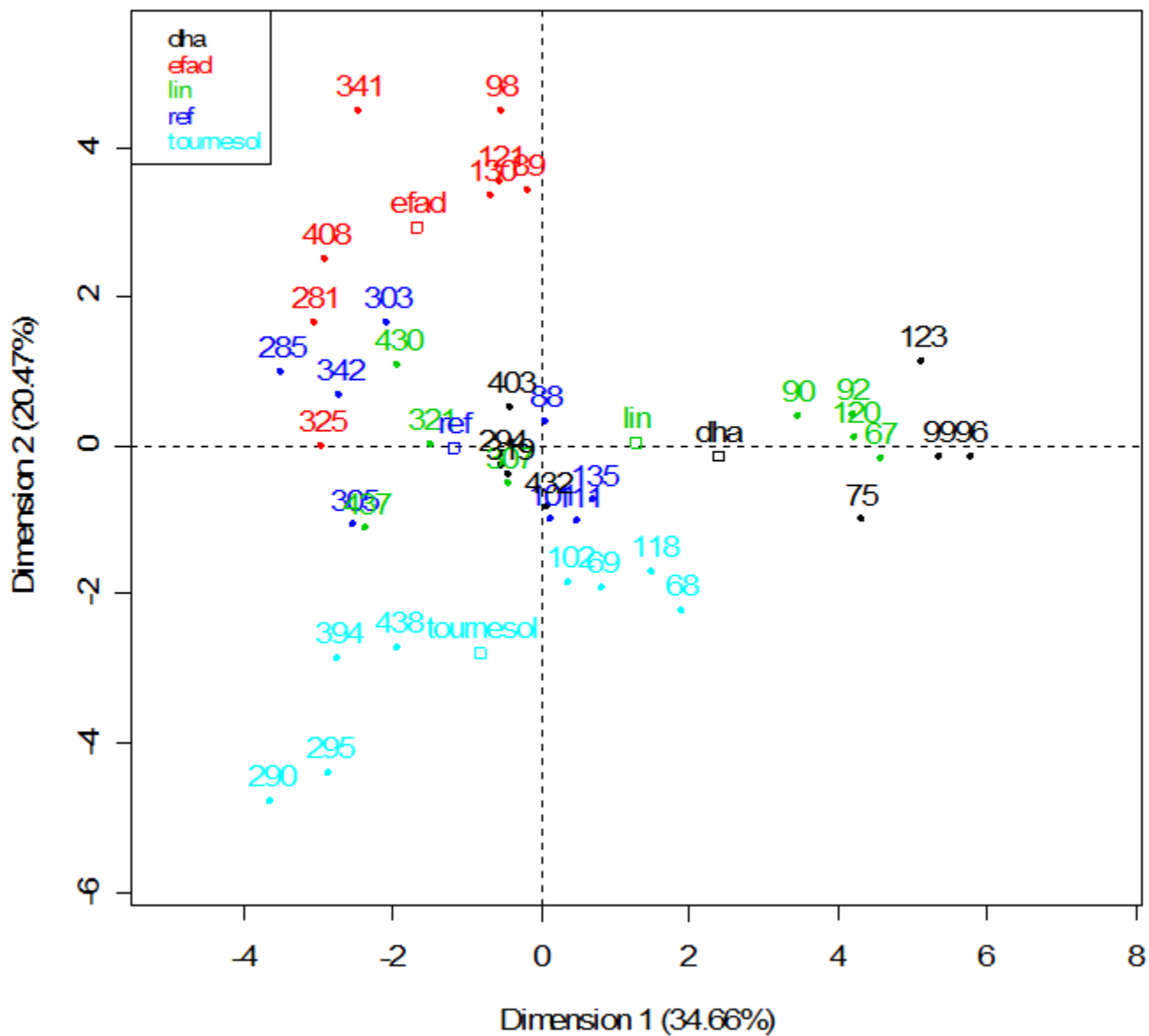
Individuals factor map (PCA)



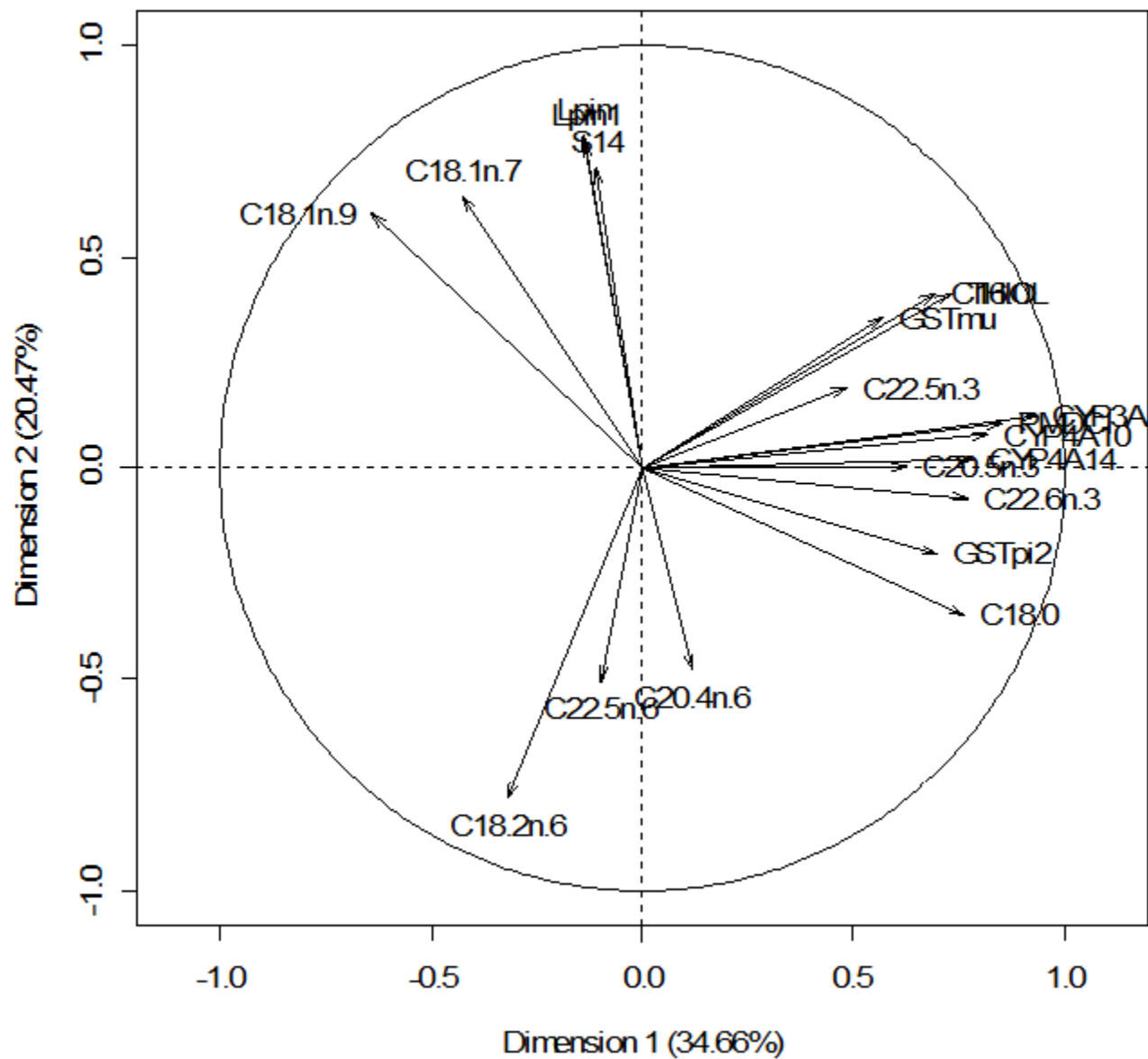
Individuals factor map (PCA)



### Individuals factor map (PCA)



Variables factor map (PCA)



# MY SECOND PCA

with supplementary qualitative and quantitative variables (and FactoMineR)

# The dataset(s)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Souris	Regime	Genotype	C14.0	C16.0	C18.0	C16.1n.9	C16.1n.7	C18.1n.9	X36b4	ACAT1	ACAT2	ACBP	ACC1	ACC2	ACO
2	67	lin	1	0.34	26.45	10.22	0.35	3.10	16.98	-0.42	-0.65	-0.84	-0.34	-1.29	-1.13	-0.93
3	68	toumesol	1	0.38	24.04	9.93	0.55	2.54	20.07	-0.44	-0.68	-0.91	-0.32	-1.23	-1.06	-0.99
4	69	toumesol	1	0.36	23.70	8.96	0.55	2.65	22.89	-0.48	-0.74	-1.1	-0.46	-1.3	-1.09	-1.06
5	75	dha	1	0.22	25.48	8.14	0.49	2.82	21.92	-0.45	-0.69	-0.65	-0.41	-1.26	-1.09	-0.93
6	88	ref	1	0.37	24.80	9.63	0.46	2.85	21.38	-0.42	-0.71	-0.54	-0.38	-1.21	-0.89	
7	89	efad	1	1.70	26.04	6.59	0.66	7.26	28.23	-0.43	-0.69	-0.8	-0.32	-1.13	-0.79	-0.93
8	90	lin	1	0.35	25.94	9.68	0.36	3.60	17.62	-0.53	-0.62	-1	-0.44	-1.22		-1
9	92	lin	1	0.34	28.63	9.95	0.29	3.27	17.02	-0.49	-0.69	-0.91	-0.37	-1.29	-1.06	-1.05
10	96	dha	1	0.22	25.34	8.81	0.44	2.36	18.39	-0.36	-0.66	-0.74	-0.39	-1.15	-1.08	-0.88
11	98	efad	1	1.38	28.49	5.63	0.90	7.01	36.68	-0.5	-0.62	-0.79	-0.36	-1.21	-0.82	-0.92
12	99	dha	1	0.26	25.73	8.30	0.43	2.74	21.75	-0.4	-0.6	-0.55	-0.25	-1.22	-1.13	-0.81
13	101	ref	1	0.44	24.28	8.63	0.53	3.33	23.86	-0.52	-0.66	-0.66	-0.41	-1.28	-1.1	-0.95
14	102	toumesol	1	0.32	24.63	9.99	0.45	2.39	17.93	-0.52	-0.63	-0.99	-0.43	-1.24	-0.96	-0.96
15	111	ref	1	0.34	26.04	9.81	0.35	2.36	20.14	-0.47	-0.71	-0.44	-0.45	-1.44	-1.17	-1.02
16	118	toumesol	1	0.35	24.76	9.38	0.45	2.47	19.66	-0.42	-0.66	-0.88	-0.38	-1.24	-1.03	-0.92
17	120	lin	1	0.24	26.46	10.97	0.41	2.81	14.69	-0.58	-0.62	-0.85	-0.32	-1.33	-1.19	-0.91
18	121	efad	1	1.21	23.45	5.59	0.65	6.1	33.84	-0.46	-0.69	-0.45	-0.32	-1.21	-0.93	-1.06
19	123	dha	1	0.30	29.72	8.95	0.45	2.6	17.79	-0.36	-0.58	-0.71	-0.34	-1.2	-0.98	-0.92
20	130	efad	1	1.30	27.00	5.72	0.81	7.6	33.50	-0.35	-0.62	-0.56	-0.24	-1.2	-0.88	-0.95
21	135	ref	1	0.38	24.09	8.22	0.60	3.09	24.61	-0.44	-0.75	-0.7	-0.33	-1.33	-1.04	-0.99
22	281	efad	2	3.24	23.59	2.68	1.11	13.09	35.61	-0.48	-0.71	-0.63	-0.55	-1.27	-1.03	-0.88
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27	303	ref	2	0.64	20.49	2.71	1.09	4.05	38.32	-0.35	-0.52	-0.74	-0.4	-1.07	-0.83	-0.73
28	305	ref	2	0.52	18.44	5.21	0.87	3.37	32.04	-0.46	-0.69	-0.53	-0.51	-1.33	-0.99	-0.9
29	307	lin	2	0.49	17.72	6.02	0.58	3.77	21.04	-0.42	-0.56	-0.86	-0.57	-1.23	-1.04	-0.75
30	319	dha	2	0.40	21.70	6.12	0.78	2.07	22.49	-0.51	-0.7	-0.68	-0.46	-1.34	-1.13	-0.94
31	321	lin	2	0.61	16.25	4.55	0.61	5.42	21.43	-0.47	-0.74	-0.88	-0.58	-1.3	-1.03	-0.83
32	325	efad	2	3.19	22.91	3.60	0.99	13.90	33.55	-0.54	-0.64	-0.39	-0.46	-1.3	-1.03	-0.85

X

1

X

2

# The dataset(s)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Souris	Regime	Genotype	C14.0	C16.0	C18.0	C16.1n.9	C16.1n.7	C18.1n.9	X36b4	ACAT1	ACAT2	ACBP	ACC1	ACC2	ACO
2	67	lin	1	0.34	26.45	10.22	0.35	3.10	16.98	-0.42	-0.65	-0.84	-0.34	-1.29	-1.13	-0.93
3	68	toumesol	1	0.38	24.04	9.93	0.55	2.54	20.07	-0.44	-0.68	-0.91	-0.32	-1.23	-1.06	-0.99
4	69	toumesol	1	0.36	23.70	8.96	0.55	2.65	22.89	-0.48	-0.74	-1.1	-0.46	-1.3	-1.09	-1.06
5	75	dha	1	0.22	25.48	8.14	0.49	2.82	21.92	-0.45	-0.69	-0.65	-0.41	-1.26	-1.09	-0.93
6	88	ref	1	0.37	24.80	9.63	0.46	2.85	21.38	-0.42	-0.71	-0.54	-0.38	-1.21	-0.89	
7	89	efad	1	1.70	26.04	6.59	0.66	7.26	28.23	-0.43	-0.69	-0.8	-0.32	-1.13	-0.79	-0.93
8	90	lin	1	0.35	25.94	9.68	0.36	3.60	17.62	-0.53	-0.62	-1	-0.44	-1.22		-1
9	92	lin	1	0.34	28.63	9.95	0.29	3.27	17.02	-0.49	-0.69	-0.91	-0.37	-1.29	-1.06	-1.05
10	96	dha	1	0.22	25.34	8.81	0.44	2.36	18.39	-0.36	-0.66	-0.74	-0.39	-1.15	-1.08	-0.88
11	98	efad	1	1.38	28.49	5.63	0.90	7.01	36.68	-0.5	-0.62	-0.79	-0.36	-1.21	-0.82	-0.92
12	99	dha	1	0.26	25.73	8.30	0.43	2.74	21.75	-0.4	-0.6	-0.55	-0.25	-1.22	-1.13	-0.81
13	101	ref	1	0.44	24.28	8.63	0.53	3.33	23.86	-0.52	-0.66	-0.66	-0.41	-1.28	-1.1	-0.95
14	102	toumesol	1	0.32	24.63	9.99	0.45	2.39	17.93	-0.52	-0.63	-0.99	-0.43	-1.24	-0.96	-0.96
15	111	ref	1	0.34	26.04	9.81	0.35	2.36	20.14	-0.47	-0.71	-0.44	-0.45	-1.44	-1.17	-1.02
16	118	toumesol	1	0.35	24.76	9.38	0.45	2.47	19.66	-0.42	-0.66	-0.88	-0.38	-1.24	-1.03	-0.92
17	120	lin	1	0.24	26.46	10.97	0.41	2.81	14.69	-0.58	-0.62	-0.85	-0.32	-1.33	-1.19	-0.91
18	121	efad	1	1.21	23.45	5.59	0.65	6.1	33.84	-0.46	-0.69	-0.45	-0.32	-1.21	-0.93	-1.06
19	123	dha	1	0.30	29.72	8.95	0.45	2.6	17.79	-0.36	-0.58	-0.71	-0.34	-1.2	-0.98	-0.92
20	130	efad	1	1.30	27.00	5.72	0.81	7.6	33.50	-0.35	-0.62	-0.56	-0.24	-1.2	-0.88	-0.95
21	135	ref	1	0.38	24.09	8.22	0.60	3.09	24.61	-0.44	-0.75	-0.7	-0.33	-1.04	-0.99	
22	281	efad	2	3.24	23.59	2.68	1.11	13.09	35.61	-0.48	-0.71	-0.63	-0.55	-1.27	-1.03	-0.88
23	285	ref	2	0.60	19.95	3.18	1.21	4.89	35.91	-0.5	-0.69	-0.86	-0.58	-1.31	-1.06	-0.88
24	290	toumesol	2	0.38	17.64	6.99	0.74	2.58	21.23	-0.54	-0.69	-1.02	-0.51	-1.32	-1.2	-0.96
25	294	dha	2	0.44	22.73	4.71	0.75	2.27	25.10	-0.39	-0.7	-0.83	-0.47	-1.27	-1.09	-0.91
26	295	toumesol	2	0.47	14.65	4.29	0.66	2.88	23.15	-0.54	-0.63	-0.86	-0.56	-1.35	-1.17	-0.88
27	303	ref	2	0.64	20.49	2.71	1.09	4.05	38.32	-0.35	-0.52	-0.74	-0.4	-1.07	-0.83	-0.73
28	305	ref	2	0.52	18.44	5.21	0.87	3.37	32.04	-0.46	-0.69	-0.53	-0.51	-1.33	-0.99	-0.9
29	307	lin	2	0.49	17.72	6.02	0.58	3.77	21.04	-0.42	-0.56	-0.86	-0.57	-1.23	-1.04	-0.75
30	319	dha	2	0.40	21.70	6.12	0.78	2.07	22.49	-0.51	-0.7	-0.68	-0.46	-1.34	-1.13	-0.94
31	321	lin	2	0.61	16.25	4.55	0.61	5.42	21.43	-0.47	-0.74	-0.88	-0.58	-1.3	-1.03	-0.83
32	325	efad	2	3.19	22.91	3.60	0.99	13.90	33.55	-0.54	-0.64	-0.39	-0.46	-1.3	-1.03	-0.85

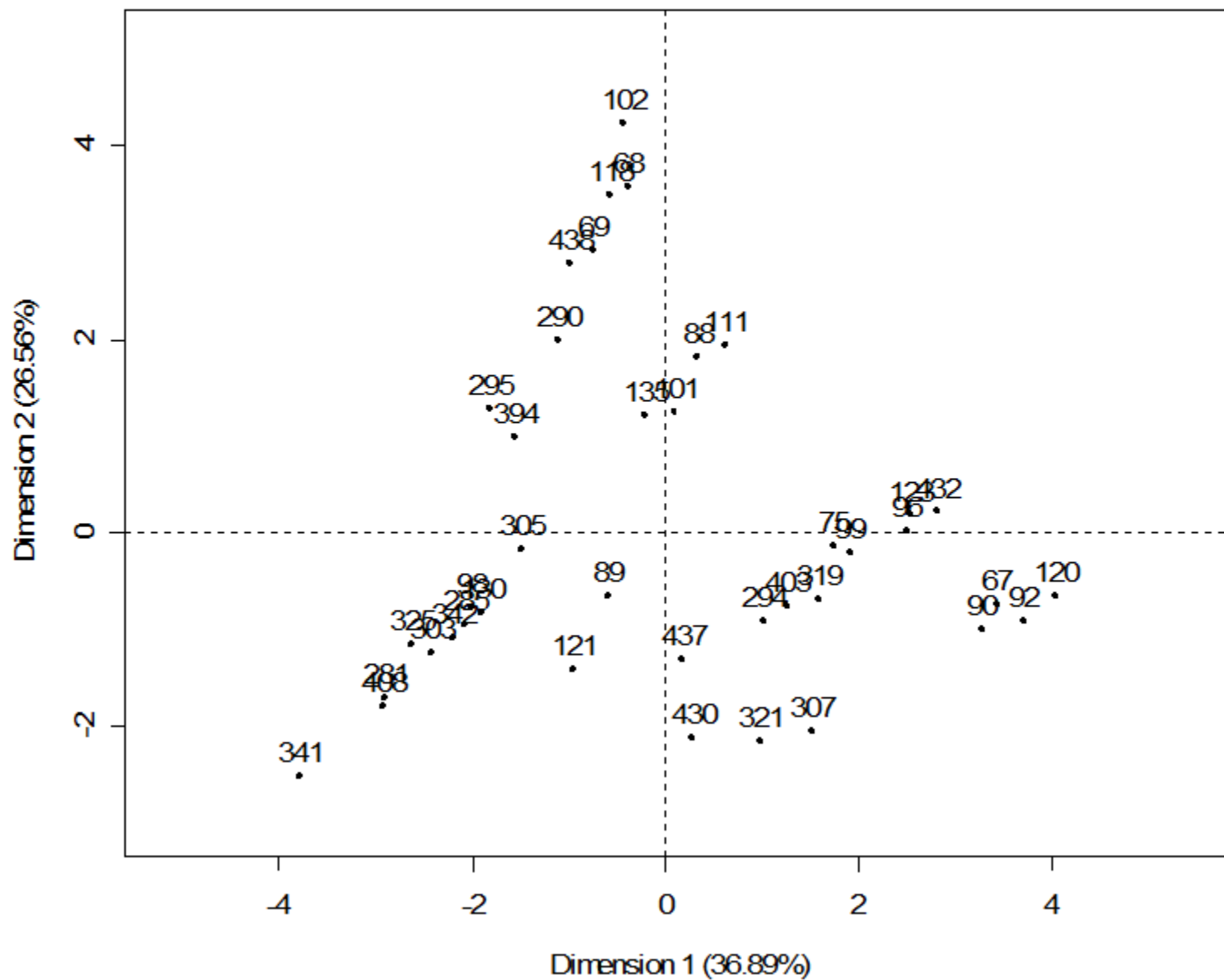
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1

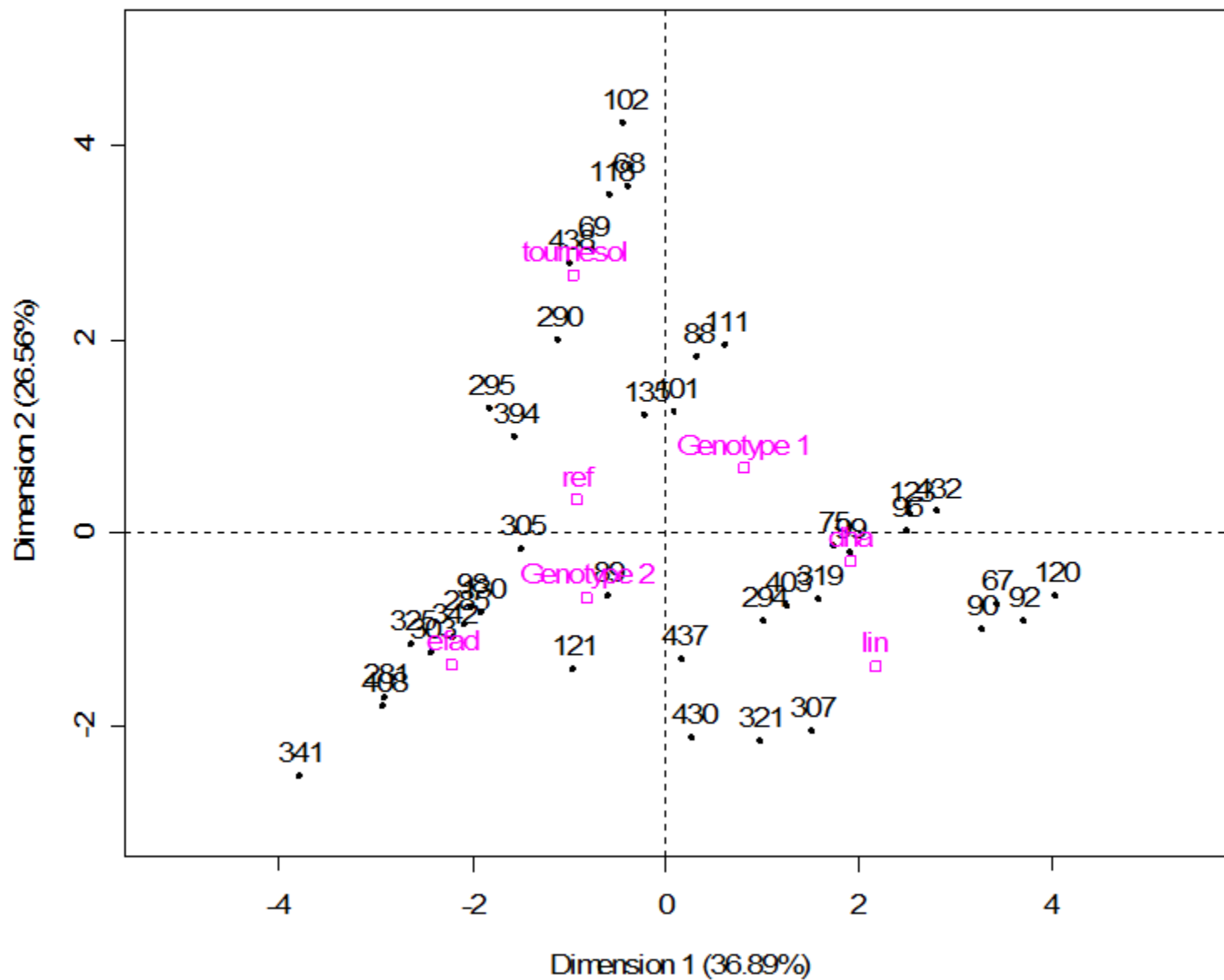
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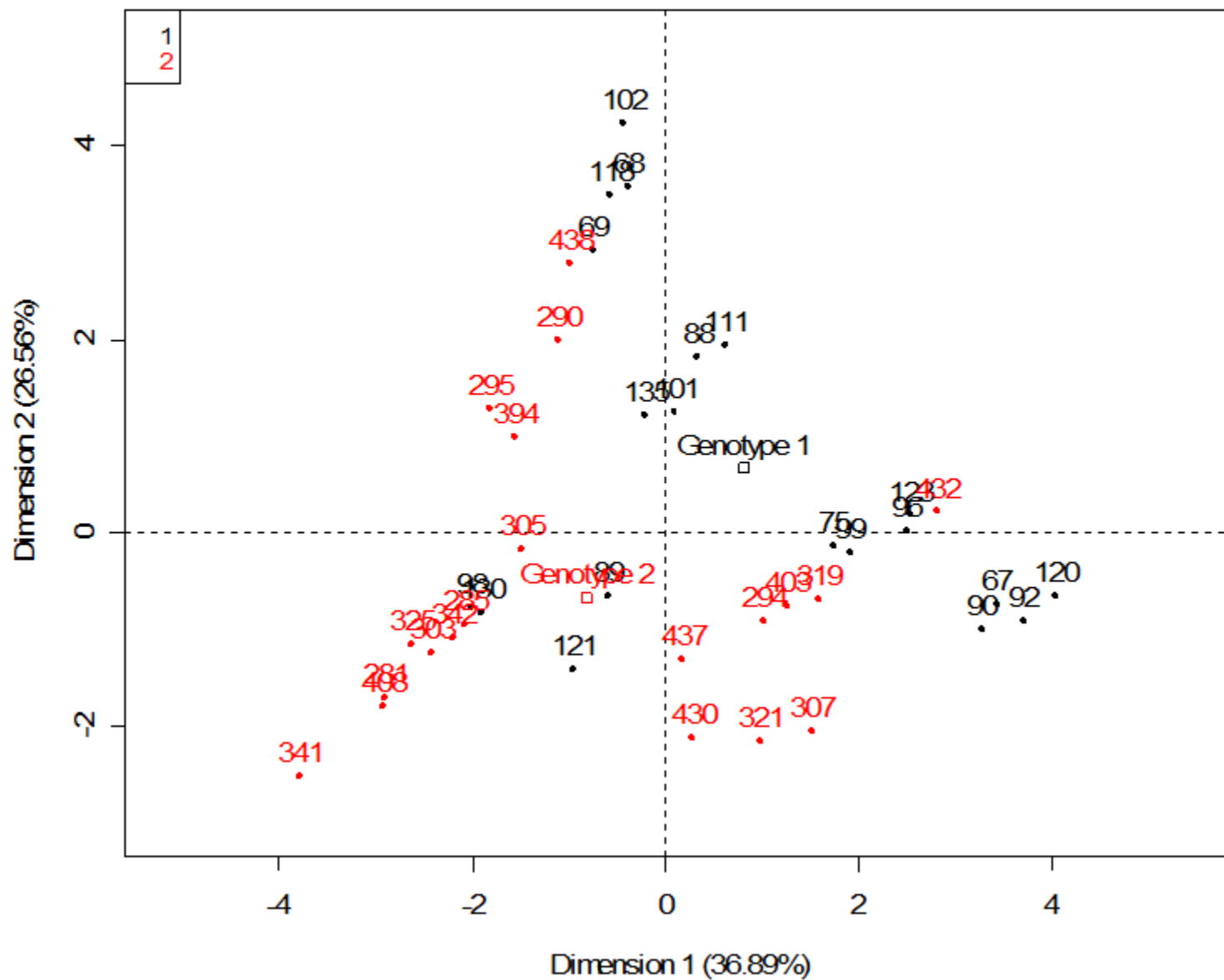
Individuals factor map (PCA)



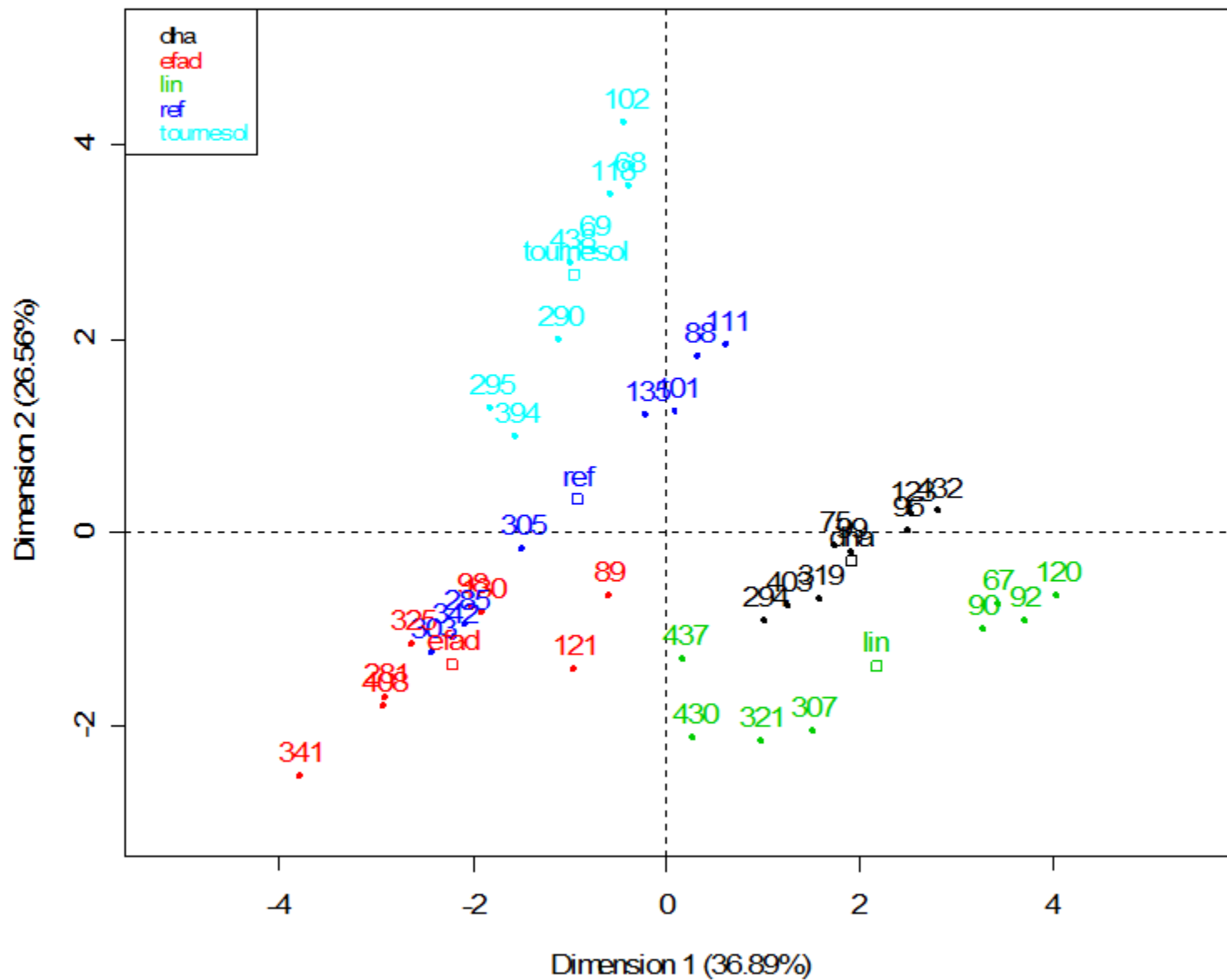
Individuals factor map (PCA)



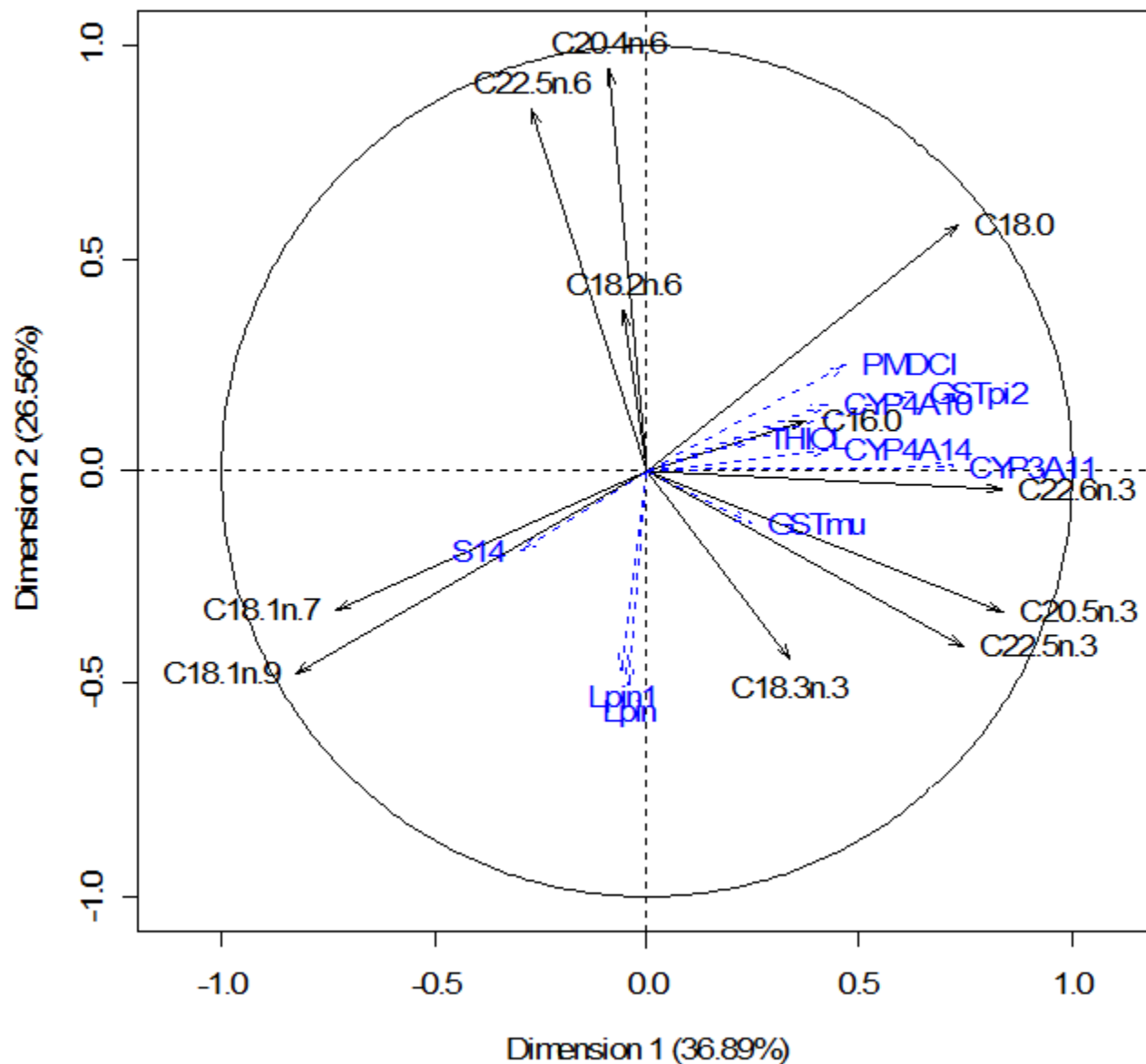
Individuals factor map (PCA)



Individuals factor map (PCA)



Variables factor map (PCA)



# The dataset(s)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Souris	Regime	Genotype	C14.0	C16.0	C18.0	C16.1n.9	C16.1n.7	C18.1n.9	X36b4	ACAT1	ACAT2	ACBP	ACC1	ACC2	ACO
2	67	lin	1	0.34	26.45	10.22	0.35	3.10	16.98	-0.42	-0.65	-0.84	-0.34	-1.29	-1.13	-0.93
3	68	toumesol	1	0.38	24.04	9.93	0.55	2.54	20.07	-0.44	-0.68	-0.91	-0.32	-1.23	-1.06	-0.99
4	69	toumesol	1	0.36	23.70	8.96	0.55	2.65	22.89	-0.48	-0.74	-1.1	-0.46	-1.3	-1.09	-1.06
5	75	dha	1	0.22	25.48	8.14	0.49	2.82	21.92	-0.45	-0.69	-0.65	-0.41	-1.26	-1.09	-0.93
6	88	ref	1	0.37	24.80	9.63	0.46	2.85	21.38	-0.42	-0.71	-0.54	-0.38	-1.21	-0.89	
7	89	efad	1	1.70	26.04	6.59	0.66	7.26	28.23	-0.43	-0.69	-0.8	-0.32	-1.13	-0.79	-0.93
8	90	lin	1	0.35	25.94	9.68	0.36	3.60	17.62	-0.53	-0.62	-1	-0.44	-1.22		-1
9	92	lin	1	0.34	28.63	9.95	0.29	3.27	17.02	-0.49	-0.69	-0.91	-0.37	-1.29	-1.06	-1.05
10	96	dha	1	0.22	25.34	8.81	0.44	2.36	18.39	-0.36	-0.66	-0.74	-0.39	-1.15	-1.08	-0.88
11	98	efad	1	1.38	28.49	5.63	0.90	7.01	36.68	-0.5	-0.62	-0.79	-0.36	-1.21	-0.82	-0.92
12	99	dha	1	0.26	25.73	8.30	0.43	2.74	21.75	-0.4	-0.6	-0.55	-0.25	-1.22	-1.13	-0.81
13	101	ref	1	0.44	24.28	8.63	0.53	3.33	23.86	-0.52	-0.66	-0.66	-0.41	-1.28	-1.1	-0.95
14	102	toumesol	1	0.32	24.63	9.99	0.45	2.39	17.93	-0.52	-0.63	-0.99	-0.43	-1.24	-0.96	-0.96
15	111	ref	1	0.34	26.04	9.81	0.35	2.36	20.14	-0.47	-0.71	-0.44	-0.45	-1.44	-1.17	-1.02
16	118	toumesol	1	0.35	24.76	9.38	0.45	2.47	19.66	-0.42	-0.66	-0.88	-0.38	-1.24	-1.03	-0.92
17	120	lin	1	0.24	26.46	10.97	0.41	2.81	14.69	-0.58	-0.62	-0.85	-0.32	-1.33	-1.19	-0.91
18	121	efad	1	1.21	23.45	5.59	0.65	6.1	33.84	-0.46	-0.69	-0.45	-0.32	-1.21	-0.93	-1.06
19	123	dha	1	0.30	29.72	8.95	0.45	2.6	17.79	-0.36	-0.58	-0.71	-0.34	-1.2	-0.98	-0.92
20	130	efad	1	1.30	27.00	5.72	0.81	7.6	33.50	-0.35	-0.62	-0.56	-0.24	-1.2	-0.88	-0.95
21	135	ref	1	0.38	24.09	8.22	0.60	3.09	24.61	-0.44	-0.75	-0.7	-0.33	-1.33	-1.04	-0.99
22	281	efad	2	3.24	23.59	2.68	1.11	13.09	35.61	-0.48	-0.71	-0.63	-0.55	-1.27	-1.03	-0.88
23	285	ref	2	0.60	19.95	3.18	1.21	4.89	35.91	-0.5	-0.69	-0.86	-0.58	-1.31	-1.06	-0.88
24	290	toumesol	2	0.38	17.64	6.99	0.74	2.58	21.23	-0.54	-0.69	-1.02	-0.51	-1.32	-1.2	-0.96
25	294	dha	2	0.44	22.73	4.71	0.75	2.27	25.10	-0.39	-0.7	-0.83	-0.47	-1.27	-1.09	-0.91
26	295	toumesol	2	0.47	14.65	4.29	0.66	2.88	23.15	-0.54	-0.63	-0.86	-0.56	-1.35	-1.17	-0.88
27	303	ref	2	0.64	20.49	2.71	1.09	4.05	38.32	-0.35	-0.52	-0.74	-0.4	-1.07	-0.83	-0.73
28	305	ref	2	0.52	18.44	5.21	0.87	3.37	32.04	-0.46	-0.69	-0.53	-0.51	-1.33	-0.99	-0.9
29	307	lin	2	0.49	17.72	6.02	0.58	3.77	21.04	-0.42	-0.56	-0.86	-0.57	-1.23	-1.04	-0.75
30	319	dha	2	0.40	21.70	6.12	0.78	2.07	22.49	-0.51	-0.7	-0.68	-0.46	-1.34	-1.13	-0.94
31	321	lin	2	0.61	16.25	4.55	0.61	5.42	21.43	-0.47	-0.74	-0.88	-0.58	-1.3	-1.03	-0.83
32	325	efad	2	3.19	22.91	3.60	0.99	13.90	33.55	-0.54	-0.64	-0.39	-0.46	-1.3	-1.03	-0.85

X

1

X

2

# The dataset(s)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Souris	Regime	Genotype	C14.0	C16.0	C18.0	C16.1n.9	C16.1n.7	C18.1n.9	X36b4	ACAT1	ACAT2	ACBP	ACC1	ACC2	ACO
2	67	lin	1	0.34	26.45	10.22	0.35	3.10	16.98	-0.42	-0.65	-0.84	-0.34	-1.29	-1.13	-0.93
3	68	toumesol	1	0.38	24.04	9.93	0.55	2.54	20.07	-0.44	-0.68	-0.91	-0.32	-1.23	-1.06	-0.99
4	69	toumesol	1	0.36	23.70	8.96	0.55	2.65	22.89	-0.48	-0.74	-1.1	-0.46	-1.3	-1.09	-1.06
5	75	dha	1	0.22	25.48	8.14	0.49	2.82	21.92	-0.45	-0.69	-0.65	-0.41	-1.26	-1.09	-0.93
6	88	ref	1	0.37	24.80	9.63	0.46	2.85	21.38	-0.42	-0.71	-0.54	-0.38	-1.21	-0.89	
7	89	efad	1	1.70	26.04	6.59	0.66	7.26	28.23	-0.43	-0.69	-0.8	-0.32	-1.13	-0.79	-0.93
8	90	lin	1	0.35	25.94	9.68	0.36	3.60	17.62	-0.53	-0.62	-1	-0.44	-1.22		-1
9	92	lin	1	0.34	28.63	9.95	0.29	3.27	17.02	-0.49	-0.69	-0.91	-0.37	-1.29	-1.06	-1.05
10	96	dha	1	0.22	25.34	8.81	0.44	2.36	18.39	-0.36	-0.66	-0.74	-0.39	-1.15	-1.08	-0.88
11	98	efad	1	1.38	28.49	5.63	0.90	7.01	36.68	-0.5	-0.62	-0.79	-0.36	-1.21	-0.82	-0.92
12	99	dha	1	0.26	25.73	8.30	0.43	2.74	21.75	-0.4	-0.6	-0.55	-0.25	-1.22	-1.13	-0.81
13	101	ref	1	0.44	24.28	8.63	0.53	3.33	23.86	-0.52	-0.66	-0.66	-0.41	-1.28	-1.1	-0.95
14	102	toumesol	1	0.32	24.63	9.99	0.45	2.39	17.93	-0.52	-0.63	-0.99	-0.43	-1.24	-0.96	-0.96
15	111	ref	1	0.34	26.04	9.81	0.35	2.36	20.14	-0.47	-0.71	-0.44	0.45	-1.44	-1.17	-1.02
16	118	toumesol	1	0.35	24.76	9.38	0.45	2.47	19.66	-0.42	-0.66	-0.88	-0.38	-1.24	-1.03	-0.92
17	120	lin	1	0.24	26.46	10.97	0.41	2.81	14.69	-0.58	-0.62	-0.85	-0.37	-1.33	-1.19	-0.91
18	121	efad	1	1.21	23.45	5.59	0.65	6.1	33.84	-0.46	-0.69	-0.45	0.32	-1.51	-0.93	-1.06
19	123	dha	1	0.30	29.72	8.95	0.45	2.6	17.79	-0.36	-0.58	-0.71	-0.34	-1.2	-0.98	-0.92
20	130	efad	1	1.30	27.00	5.72	0.81	7.6	33.50	-0.35	-0.62	-0.56	-0.24	-1.27	-0.88	-0.95
21	135	ref	1	0.38	24.09	8.22	0.60	3.09	24.61	-0.44	-0.75	-0.7	-0.33	-1.33	-1.04	-0.99
22	281	efad	2	3.24	23.59	2.68	1.11	13.09	35.61	-0.48	-0.71	-0.63	-0.55	-1.27	-1.03	-0.88
23	285	ref	2	0.60	19.95	3.18	1.21	4.89	35.91	-0.5	-0.69	-0.86	-0.58	-1.31	-1.06	-0.88
24	290	toumesol	2	0.38	17.64	6.99	0.74	2.58	21.23	-0.54	-0.69	-1.02	-0.51	-1.32	-1.2	-0.96
25	294	dha	2	0.44	22.73	4.71	0.75	2.27	25.10	-0.39	-0.7	-0.83	-0.47	-1.27	-1.09	-0.91
26	295	toumesol	2	0.47	14.65	4.29	0.66	2.88	23.15	-0.54	-0.63	-0.86	-0.56	-1.35	-1.17	-0.88
27	303	ref	2	0.64	20.49	2.71	1.09	4.05	38.32	-0.35	-0.52	-0.74	-0.4	-1.07	-0.83	-0.73
28	305	ref	2	0.52	18.44	5.21	0.87	3.37	32.04	-0.46	-0.69	-0.53	-0.51	-1.33	-0.99	-0.9
29	307	lin	2	0.49	17.72	6.02	0.58	3.77	21.04	-0.42	-0.56	-0.86	-0.57	-1.23	-1.04	-0.75
30	319	dha	2	0.40	21.70	6.12	0.78	2.07	22.49	-0.51	-0.7	-0.68	-0.46	-1.34	-1.13	-0.94
31	321	lin	2	0.61	16.25	4.55	0.61	5.42	21.43	-0.47	-0.74	-0.88	-0.58	-1.3	-1.03	-0.83
32	325	efad	2	3.19	22.91	3.60	0.99	13.90	33.55	-0.54	-0.64	-0.39	-0.46	-1.3	-1.03	-0.85

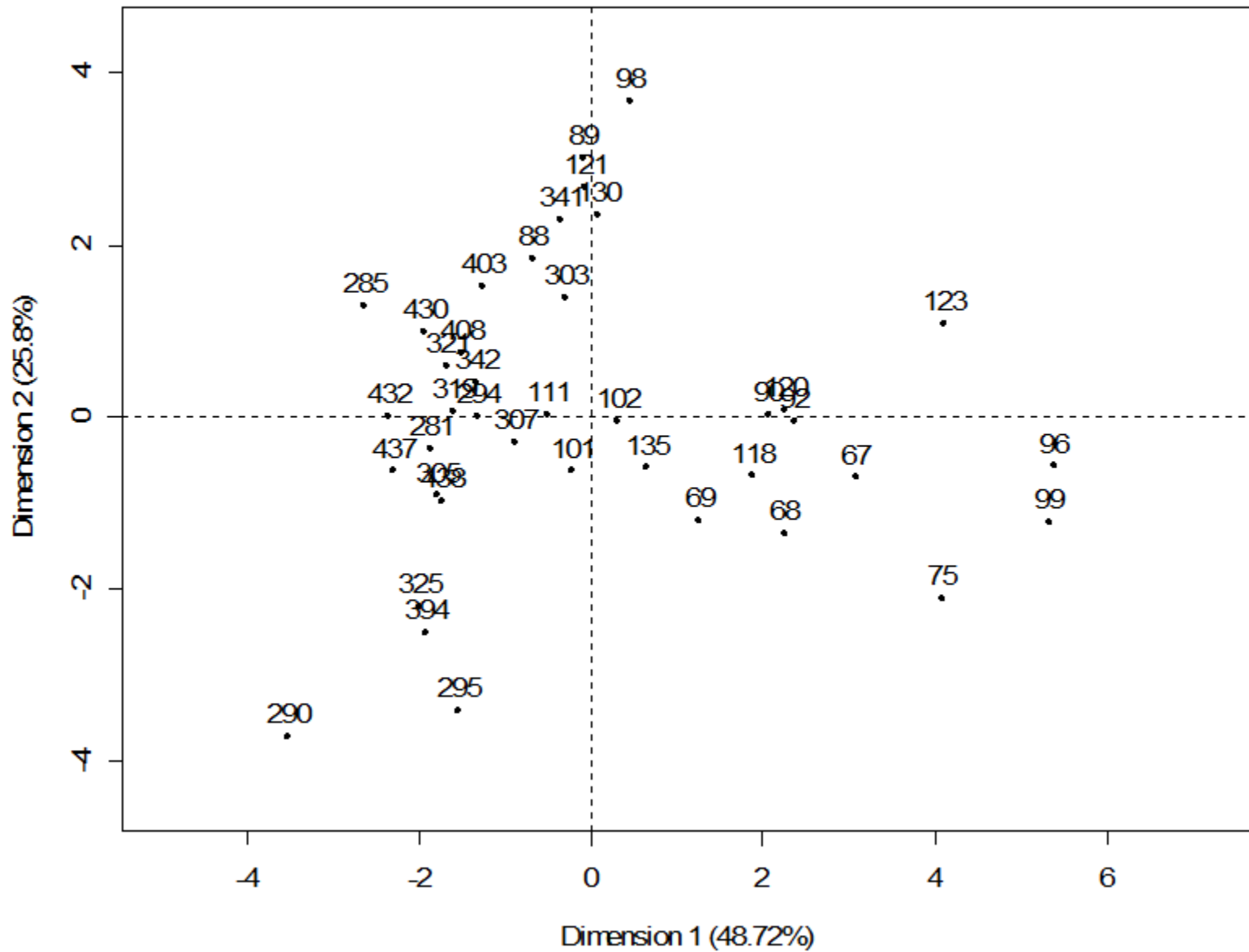
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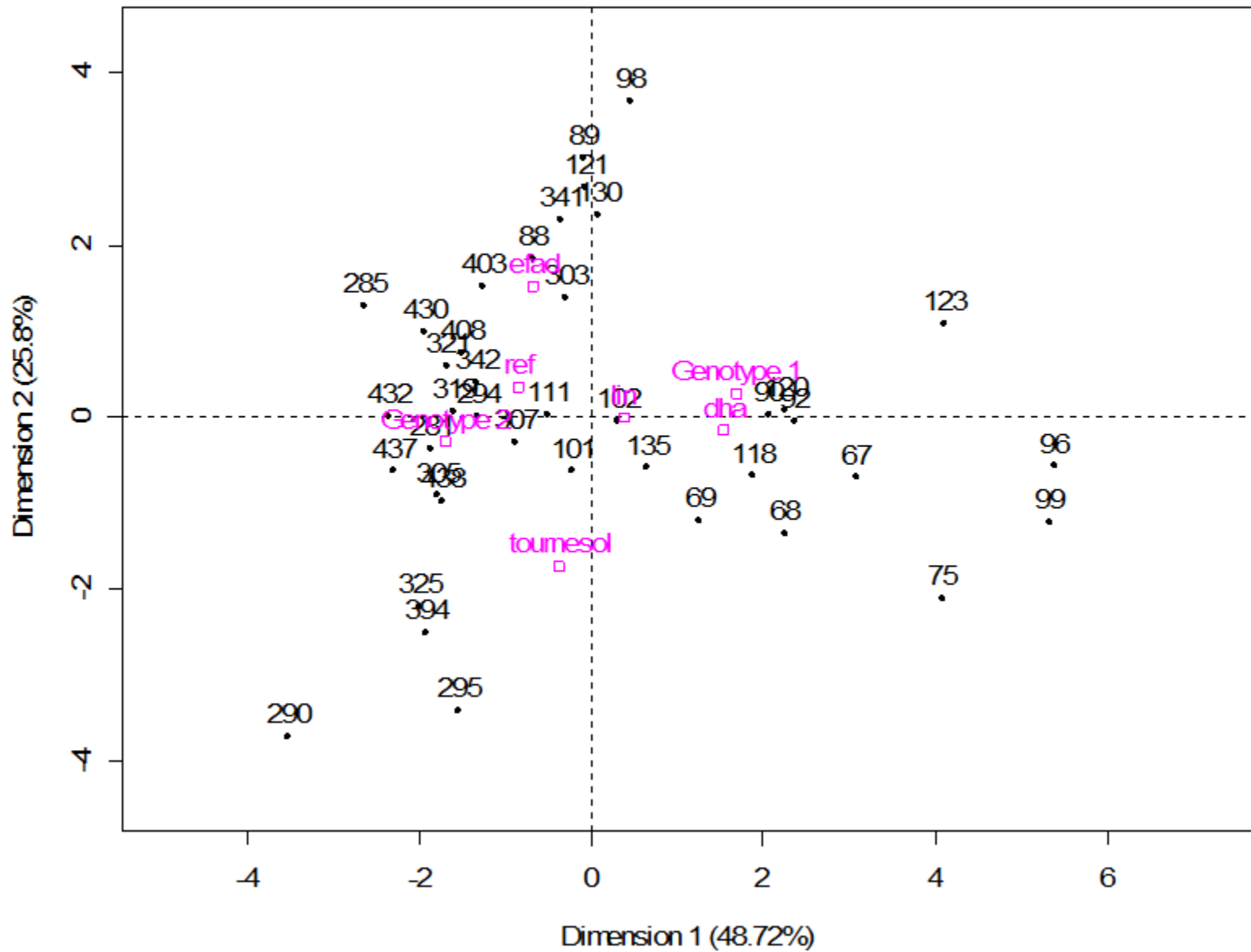
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2

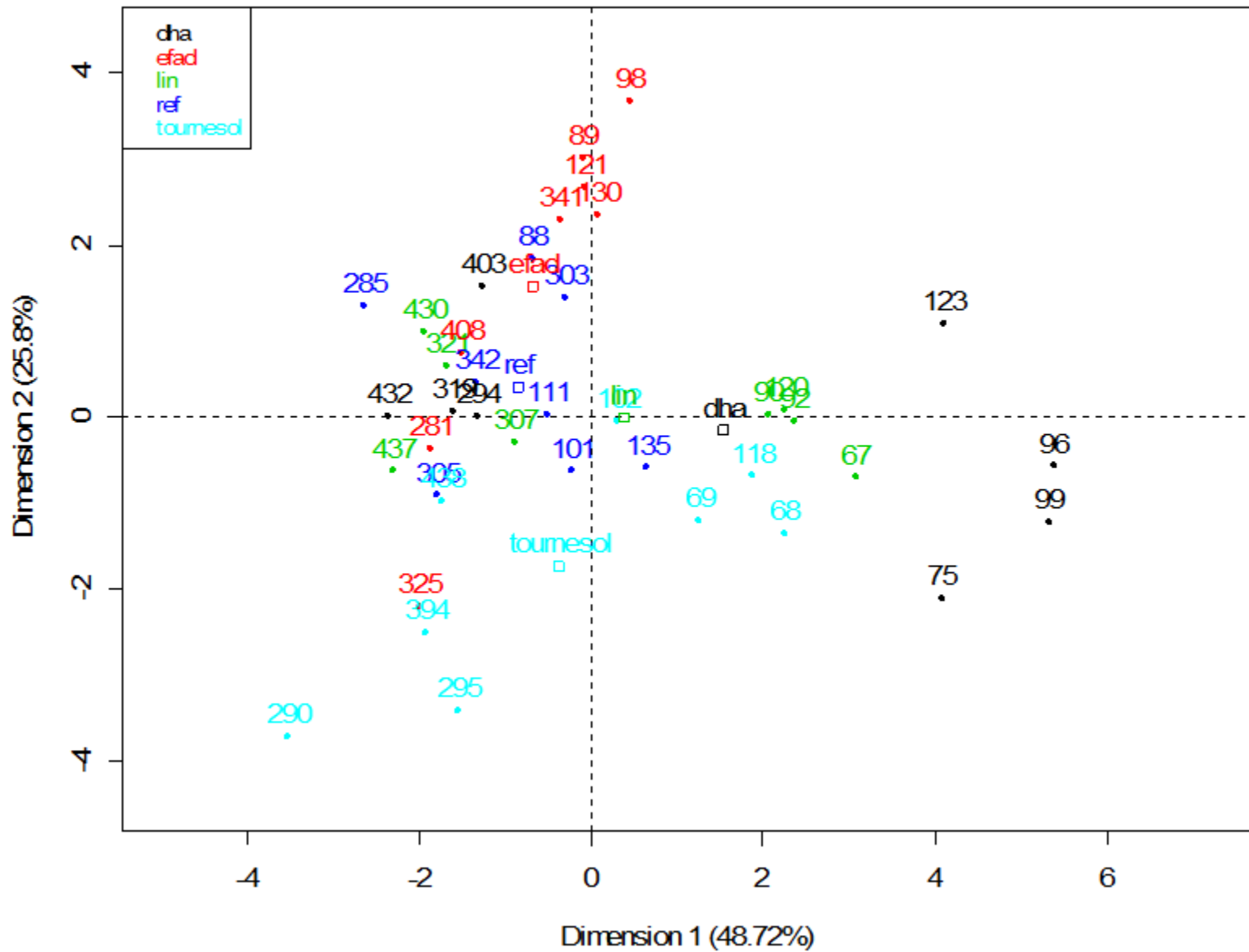
Individuals factor map (PCA)



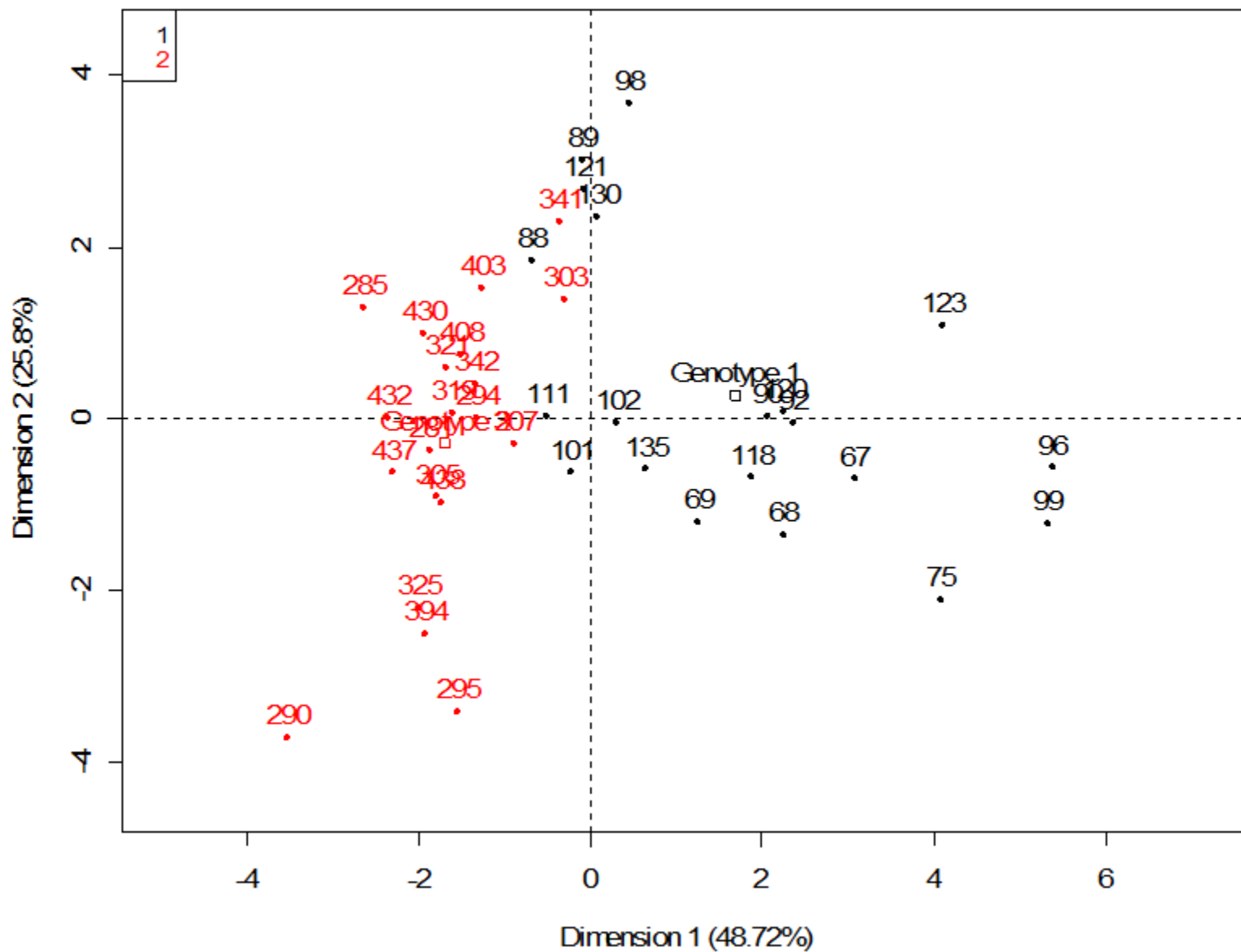
Individuals factor map (PCA)



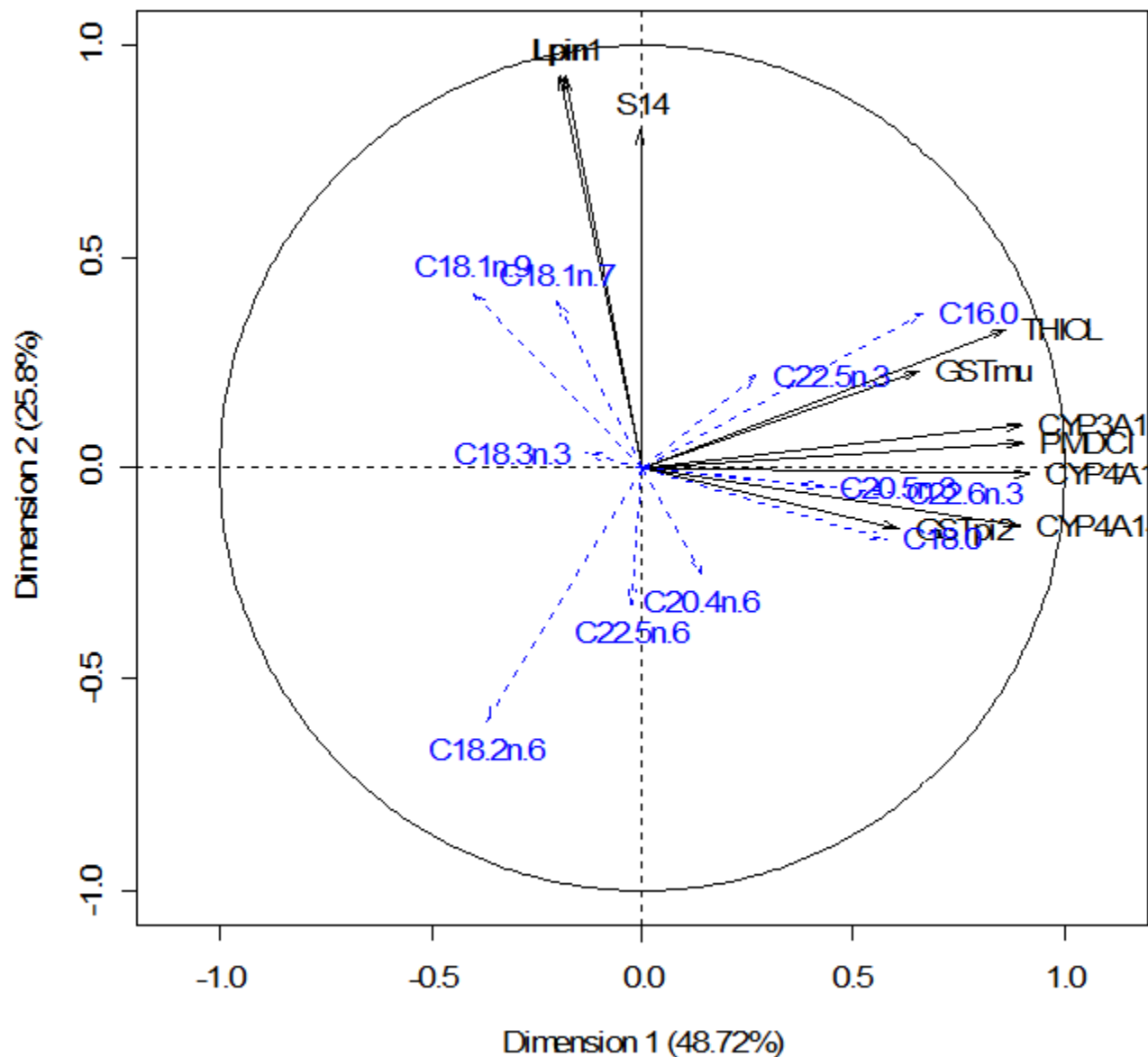
### Individuals factor map (PCA)



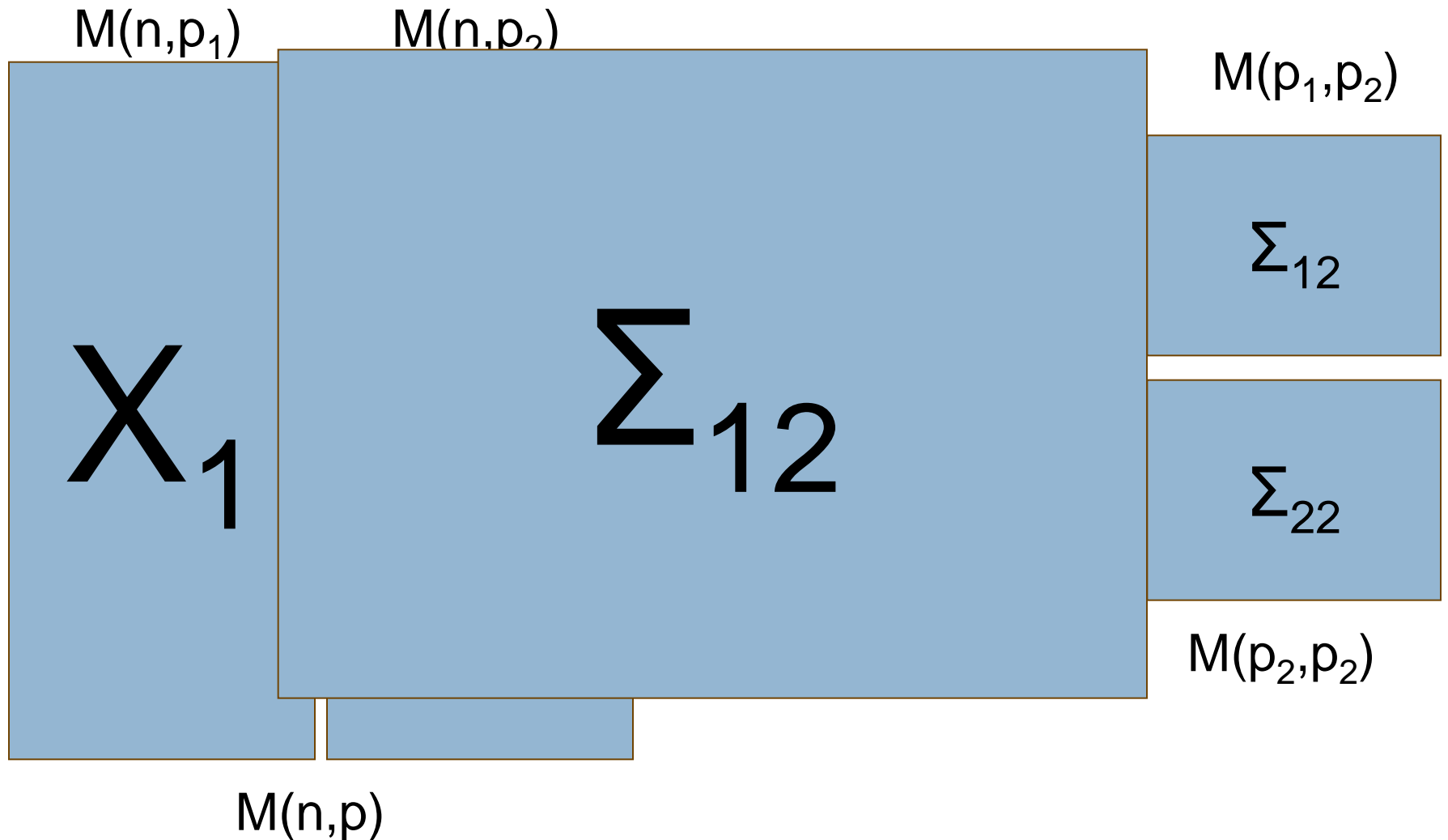
Individuals factor map (PCA)



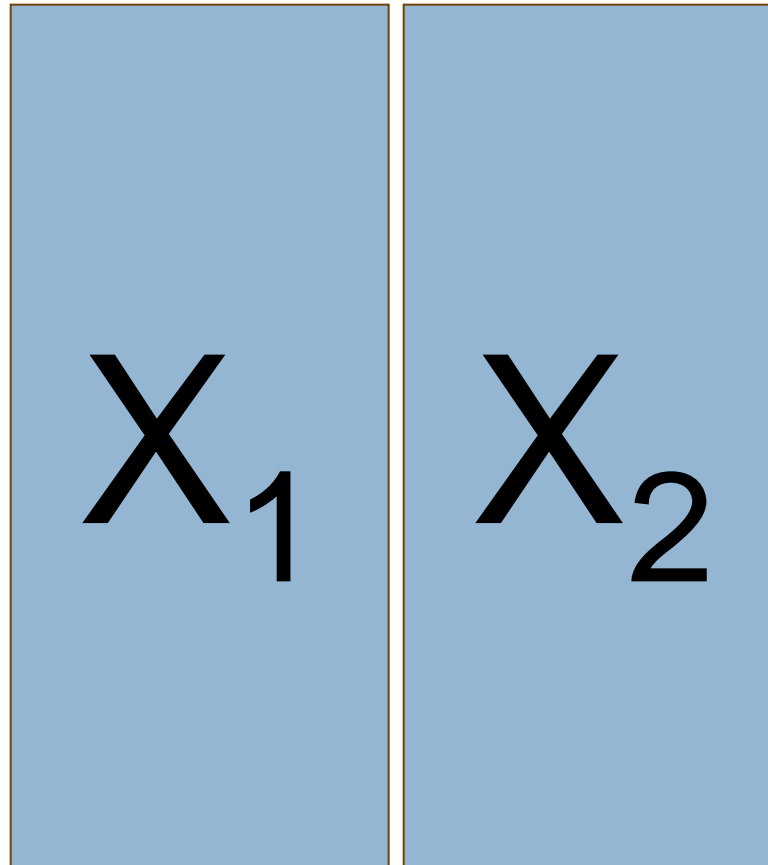
Variables factor map (PCA)



# The dataset(s)



# Close Encounters of the Third Kind



*FROM MULTIVARIATE TO  
MULTIPLE TABLES DATA  
ANALYSIS*



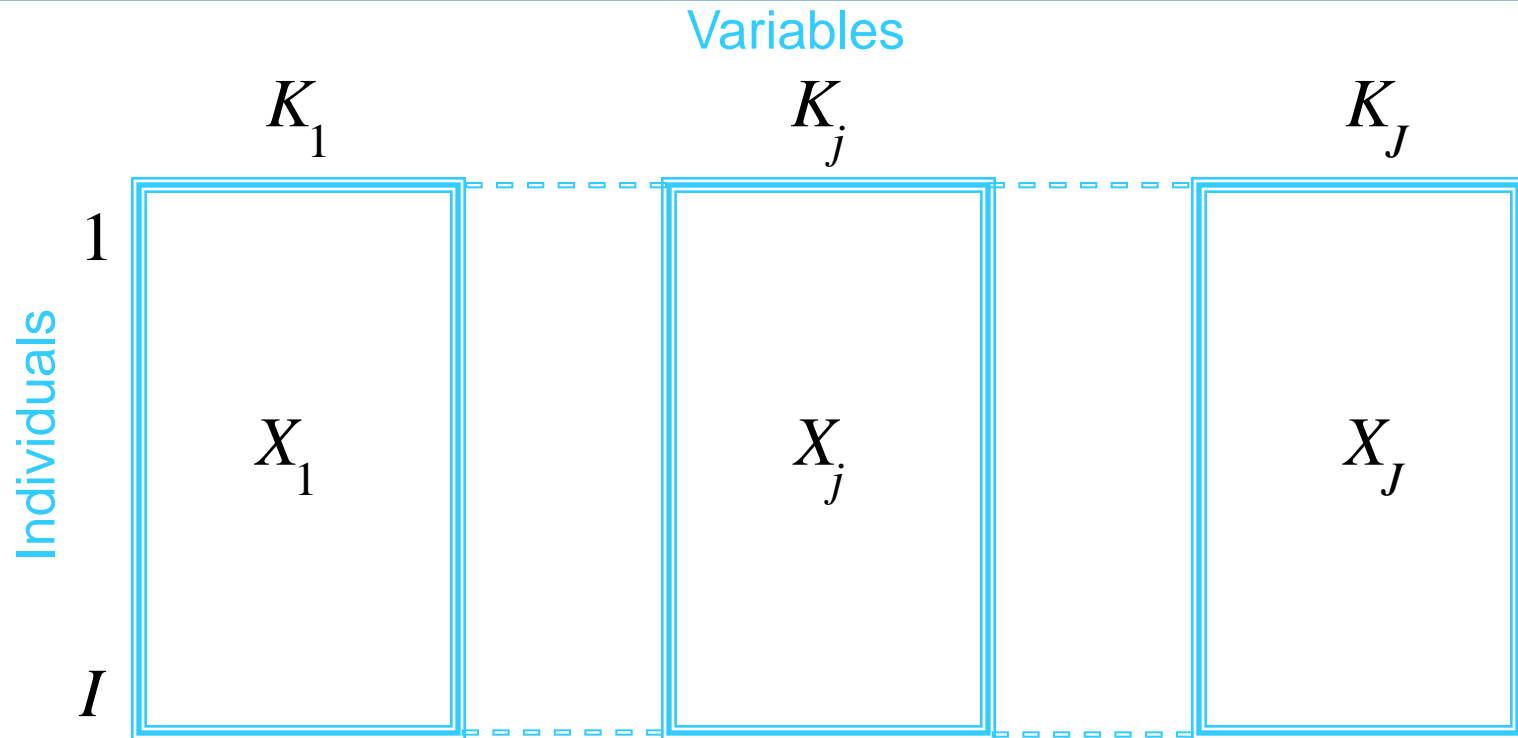
# Rashomon (1950, Kurosawa)



# Rashomon (1950, Kurosawa)

- The film depicts the rape of a woman and the apparent murder of her husband through the widely differing accounts of **four witnesses**, including the rapist and, through a medium, the dead man.
- The stories are mutually contradictory, leaving the viewer to determine which, if any, is the truth.

# Objectives underlying the study of several groups of variables



# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- Comparison of factors
- Overall representation of groups
- Superimposed representation

# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- Comparison of factors
- Overall representation of groups
- Superimposed representation

To balance the influence of each group in a simultaneous analysis

# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- Comparison of factors
- Overall representation of groups
- Superimposed representation

To search for factors that are common to the group of variables

# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- **Comparison of factors**
- Overall representation of groups
- Superimposed representation

**To compare the factors of several groups of variables**

# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- Comparison of factors
- Overall representation of groups
- Superimposed representation

Two groups are all the more close that they induce the same structure

# Objectives underlying the study of several groups of variables

- Weighting of the variables
- Looking for common factors
- Comparison of factors
- Overall representation of groups
- Superimposed representation

An individual is all the more “homogenous” that its superimposed representations are close

# WEIGHTING OF THE VARIABLES

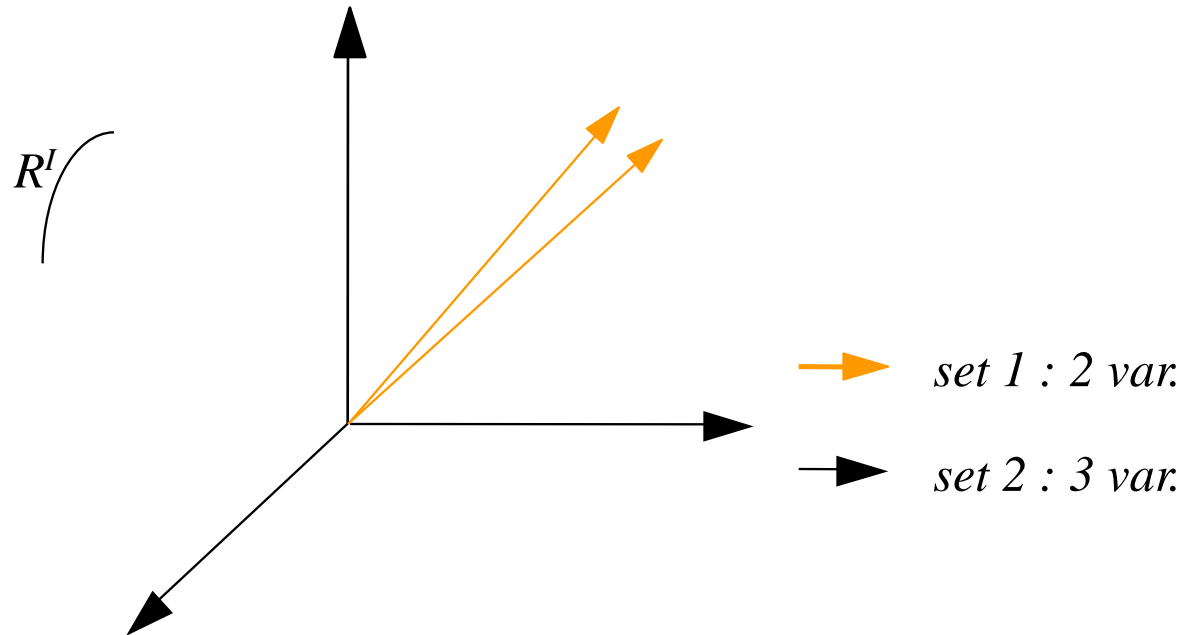


# On the interest of balancing the influence of each group of variables

- By analogy with the individuals: weighting sample surveys, balanced data
- Same weight for each variable of a given group
  - Number of variables
  - Structure of each group

# On the interest of balancing the influence of each group of variables

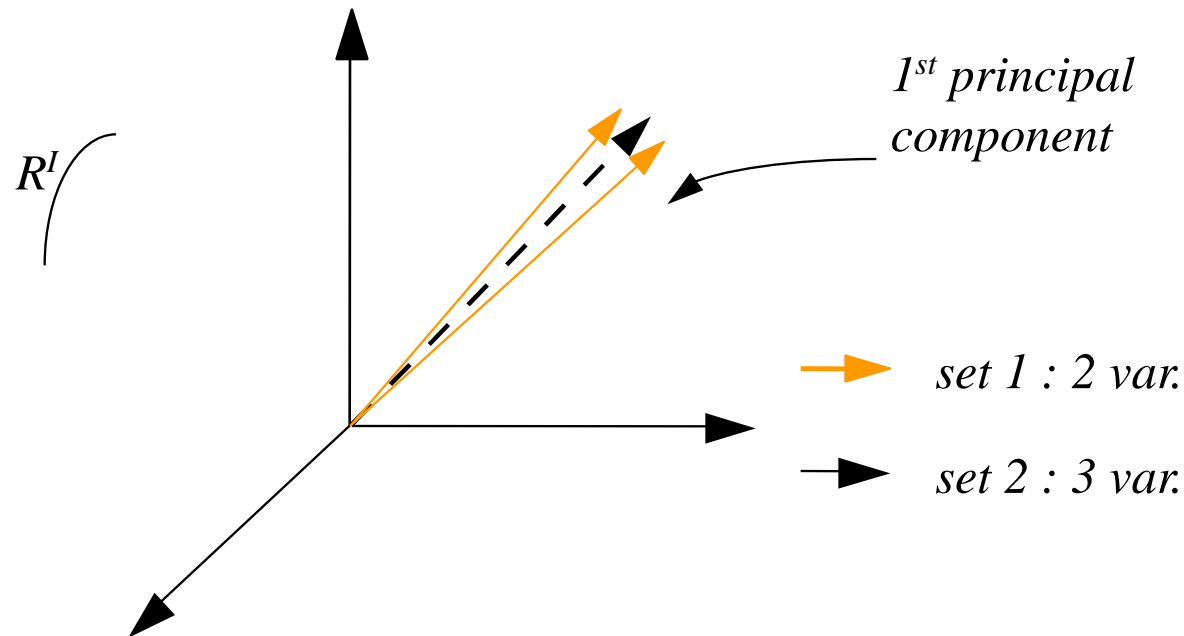
## Reference example



# On the interest of balancing the influence of each group of variables

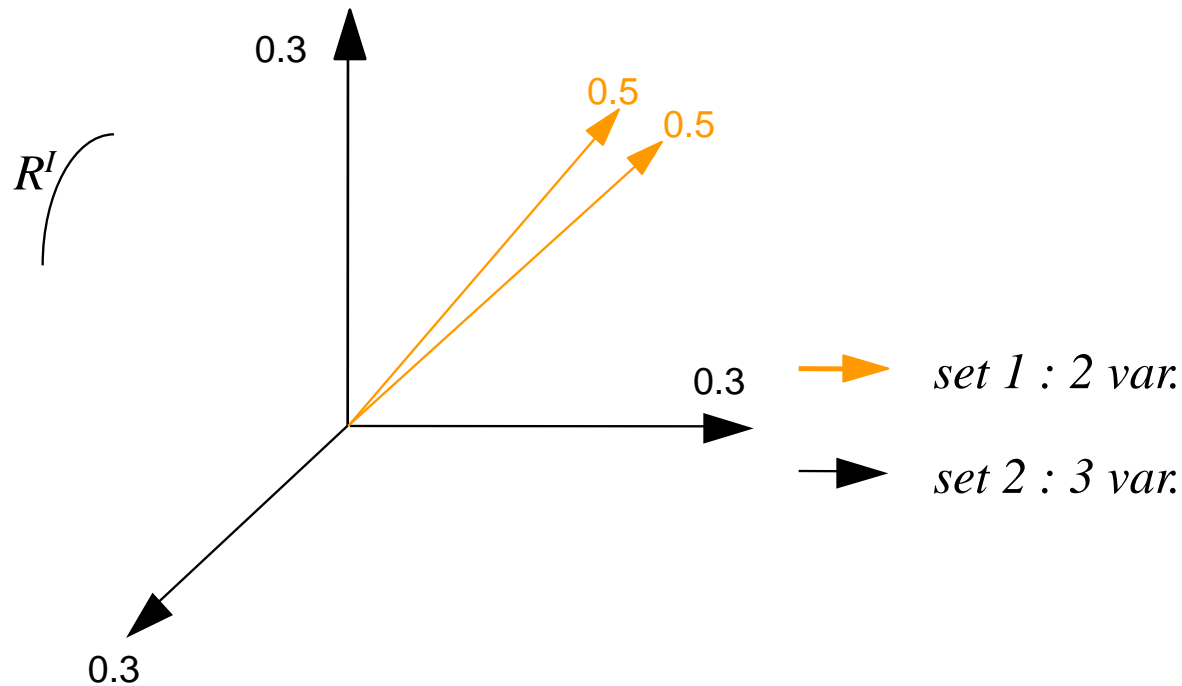
## Reference example

PCA of the 5 variables, without considering the sets



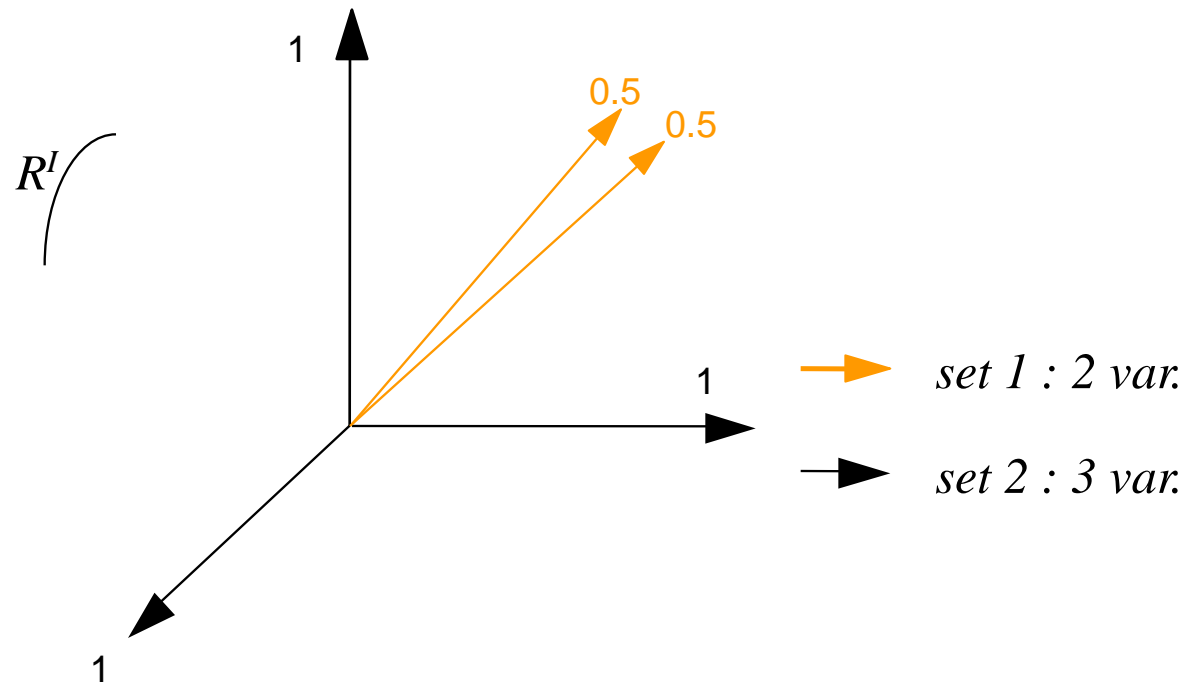
# On the interest of balancing the influence of each group of variables

## Reference example Balancing the sets by the total “inertia”



# On the interest of balancing the influence of each group of variables

## Reference example Balancing the sets of variables in MFA



Each variable of the set  $j$  is weighted by  $1/\lambda_1^j$   
 $\lambda_1^j$ : 1<sup>st</sup> eigenvalue of PCA applied to set  $j$ .

## On the interest of balancing the influence of each group of variables

- For each group the variance of the main axis of variability is equal to 1
- No group can generate all by itself the first global axis
- A “multidimensional” group will contribute to the construction of more axes than a “one-dimensional” group
- This weighting is a specific characteristic of MFA; it induces many properties described later

# Weighted factorial analysis



MFA is based on a “factorial analysis” applied to all active sets of variables

# Weighted factorial analysis



MFA is based on a “factorial analysis” applied to all active sets of variables

De facto: MFA beneficiates from the transition formulae and from the duality between *individuals* and *variables*.

# Weighted factorial analysis

MFA is based on a “factorial analysis” applied to all active sets of variables

Quantitative variables: MFA is based on a weighted PCA

standardized variables

unstandardized variables

mixed

Equivalence

When each set is composed by 1 quantitative variable:  $MFA = PCA$

# Weighted factorial analysis

MFA is based on a “factorial analysis” applied to all active sets of variables

MFA provides:

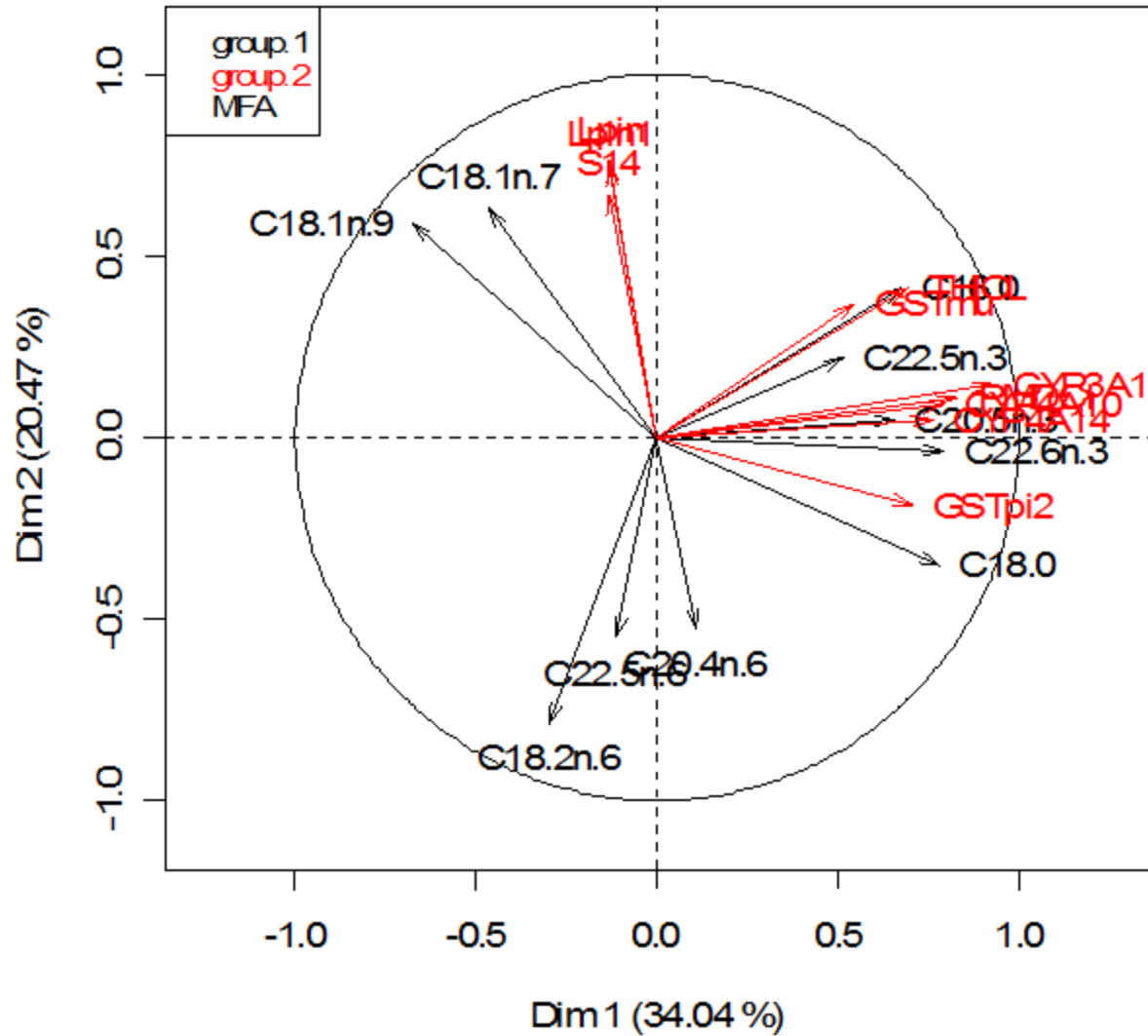
Firstly: classical results of factorial analysis

For each axis:

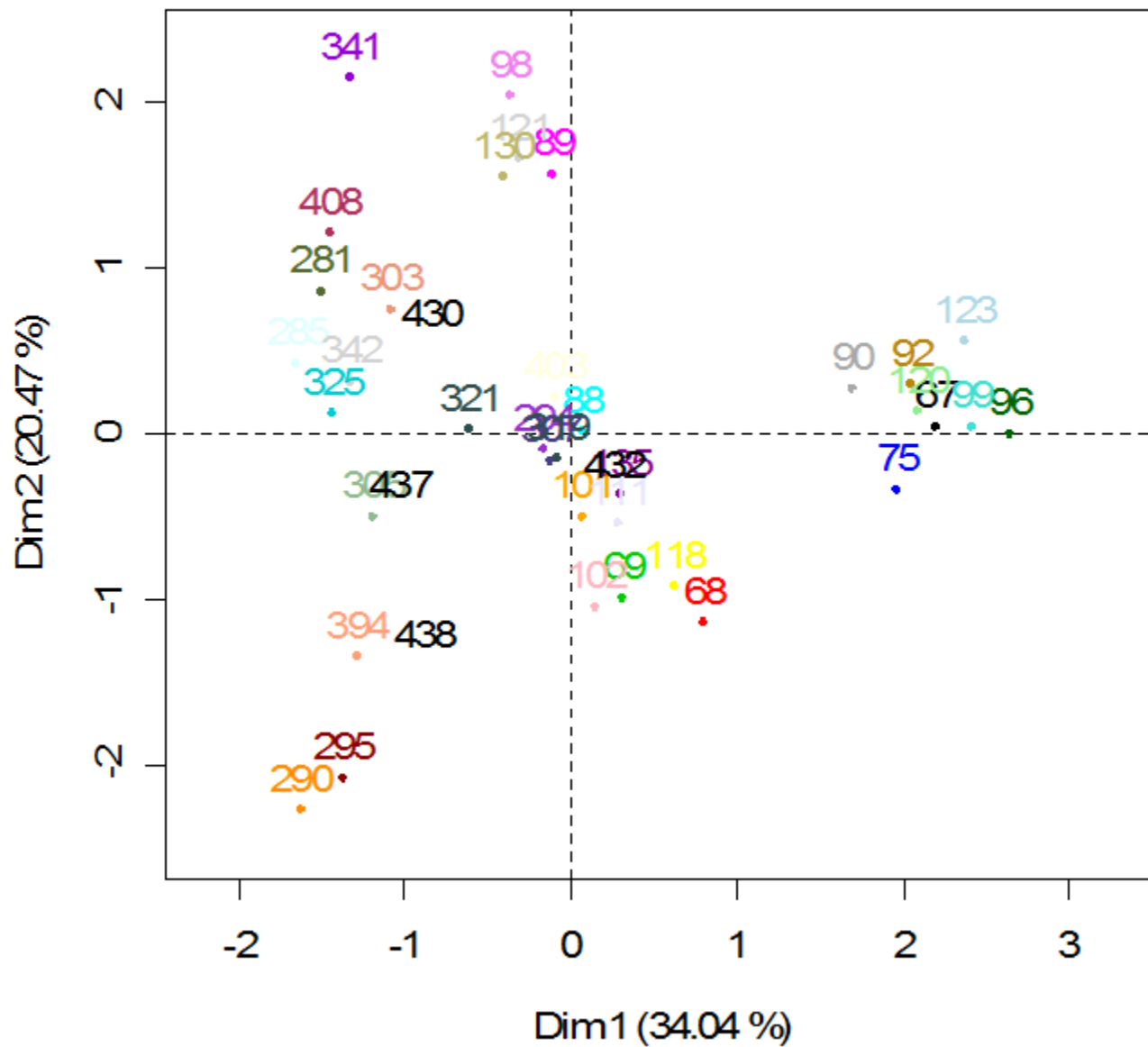
Co-ordinates, contributions and squared cosines of individuals

Correlation coefficients between factors and continuous variables

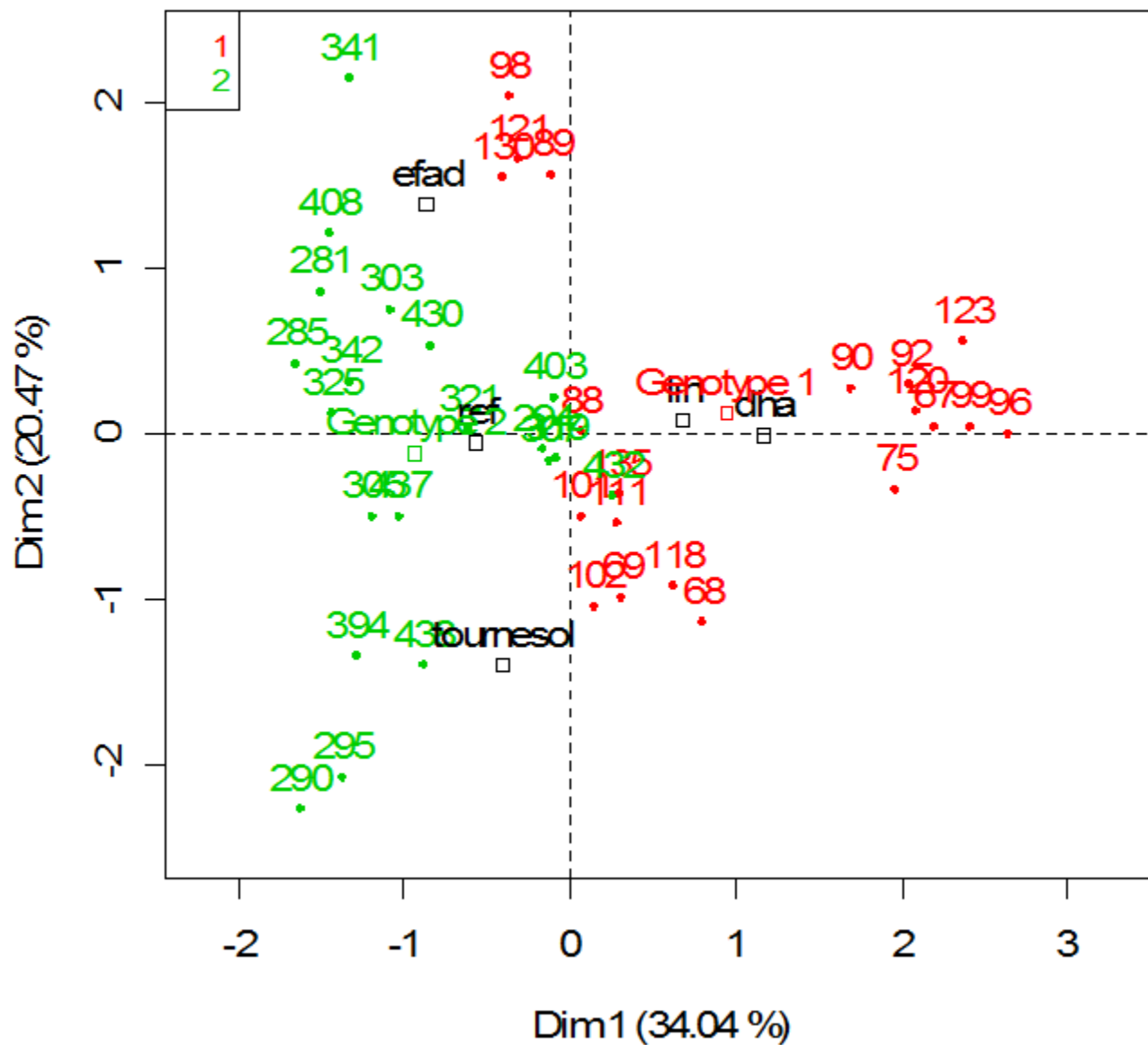
# Correlation circle



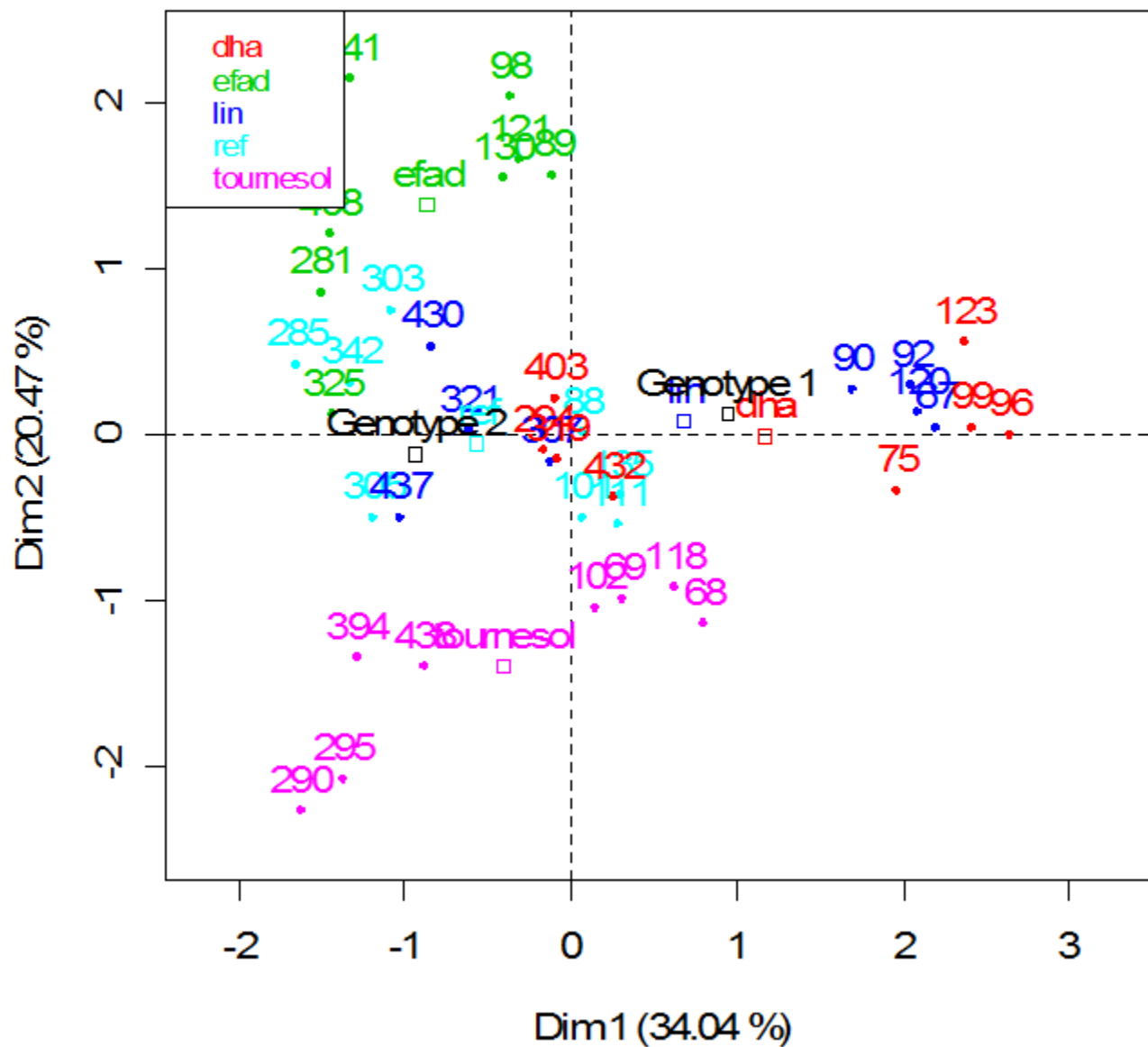
# Individual factor map



# Individual factor map



# Individual factor map





# SETTING UP COMMON FACTORS



# Looking for factors common to TWO sets of variables

$X_1$



$X_2$



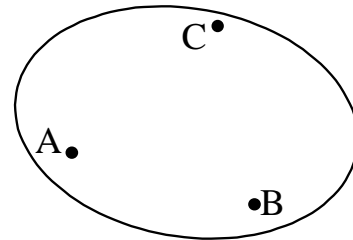
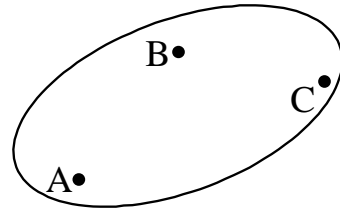
Reference method: Canonical Analysis  
Hotelling, 1936

## Looking for factors common to TWO sets of variables

- The word “**canonical**” comes from the Greek ΚΑΝΩΝ / *kanôn* that means “ruler”
- The purpose of Canonical analysis is to find the relationship between two groups of variables
- It works by finding two linear combinations of variables, one for each group, which are most highly correlated
- Hotelling, H. (1936) Relations between two sets of variables. *Biometrika*, 28, 321-377

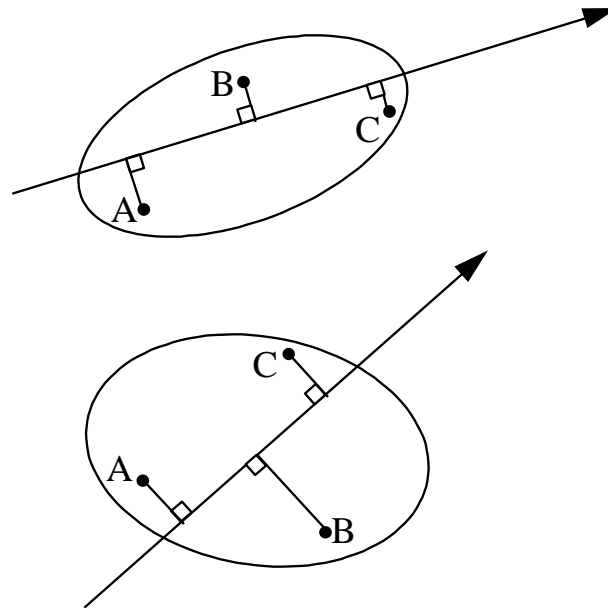
# Looking for factors common to TWO sets of variables

A factor common to two clouds?



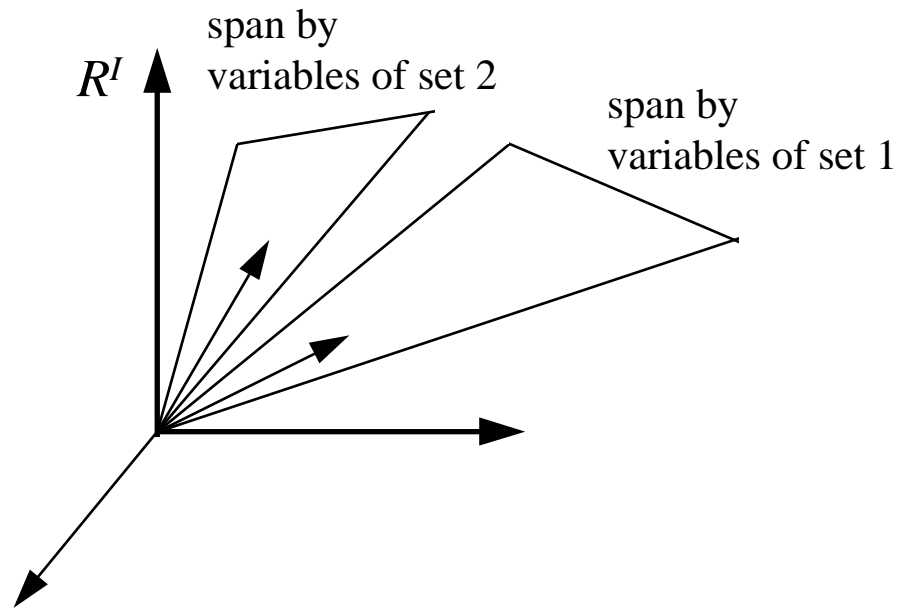
# Looking for factors common to TWO sets of variables

A factor common to two clouds!



# Looking for factors common to TWO sets of variables

Looking for jointly linear combinations of variables of sets 1 and 2



# cancor(lip,gen)

## R Console

Fichier Edition Misc Packages Aide

```
> cancor(lip,gen)
```

```
$cor
```

```
[1] 0.96297273 0.93199442 0.91149643 0.85708380 0.78971594 0.71861893 0.60593887 0.40912187 0.24543555 0.04250940
```

```
$xcoef
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	
C18.1n.9	0.028997123	0.0171655617	-0.032635011	-0.01450857	-0.018555194	-0.041183319	-0.013174087	0.040586602	0.08710232	
C18.1n.7	0.032505732	-0.0								
C18.2n.6	0.011143241	-0.0								
C20.4n.6	0.016316003	0.0								
C22.5n.6	0.013895923	-0.1994208277	-0.237792213	0.04828274	-0.294048565	-0.034069099	-0.278425025	-0.097912648	-0.01606464	
C22.6n.3	-0.009625662	0.0180690151	-0.057893604	-0.01258168	0.010209536	-0.016871470	-0.040695824	0.018336510	0.07227701	
C20.5n.3	0.001693435	0.0287109724	-0.052755724	0.11695136	0.028633355	-0.033946554	0.011660799	-0.006932460	0.11639351	
C22.5n.3	0.080104739	0.0189714987	-0.078025488	-0.09597613	0.004368761	0.114483628	0.243505319	0.029838355	0.28063148	
C16.0	0.025879989	0.0002305252	-0.014961055	-0.01606147	-0.087258413	-0.021124677	0.052003685	0.075732380	0.05434408	
C18.0	0.053045462	0.0023552992	0.060777399	-0.15890538	-0.105767029	-0.003898920	-0.111955143	0.100838476	0.11642331	
C18.3n.3	0.012796276	0.0					0.031400092	-0.011095415	0.033605112	0.09007832

Beware: canonical variables

$G_1 = 0.51\text{PMDCI} + 0.63\text{THIOL} + \dots - 0.30\text{CYP4A14}$

```
$ycoef
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
PMDCI	0.51640610	0.6523181	1.0410813	-1.17646805	-0.23554143	0.96947711	-0.52053547	-0.16848881	0.1407219	-0.5615899
THIOL	0.63929635	0.3485394	-0.5578365	1.21794331	1.00912511	-0.40821774	0.12353419	-0.25882855	0.2587056	0.0627558
CYP3A11	-0.60991186	0.0407698	-0.2250082	0.26683169	-0.40367530	-0.01861773	0.53348063	-0.90329962	-0.4644879	0.3434860
CYP4A10	-0.05180081	-0.3360889	-0.1978164	0.19342476	-0.91569517	-0.05456395	-0.49015959	1.02698256	0.7043031	1.4347729
Lpin	0.26195541	0.2501206	-0.6088044	-0.13511715	-1.10231985	-0.27382711	-0.09176426	0.13063574	3.9408262	-1.7372312
Lpin1	-0.27203283	0.335							113	2.1529151
GSTmu	0.24643161	-0.565							318	0.3644918
GSTpi2	-0.25550623	0.374							683	-0.6715756
S14	0.16336084	-0.3291935	0.1997579	0.06817508	-0.64894459	-0.01595339	0.56824837	0.05062431	-0.5990380	-0.6348794
CYP4A14	-0.30233566	0.1143903	-0.1328348	-0.33811760	0.30070528	-0.69227204	-0.18733677	-0.10210937	-0.5592082	-1.3047358

```
$xcenter
```

```
C18.1n.9 C18.1n.7 C18.2n.6 C20.4n.6 C22.5n.6 C22.6n.3 C20.5n.3 C22.5n.3 C16.0 C18.0 C18.3n.3  
25.27325 4.42600 15.27750 5.27925 0.43700 5.91400 1.78950 0.87175 23.02600 6.74700 2.88800
```

```
$ycenter
```

```
PMDCI THIOL CYP3A11 CYP4A10 Lpin Lpin1 GSTmu GSTpi2 S14 CYP4A14  
-0.76725 -0.41100 -0.50825 -0.97975 -0.75325 -0.76475 -0.11900 0.22975 -0.80675 -0.99300
```

# cancor(lip,gen)

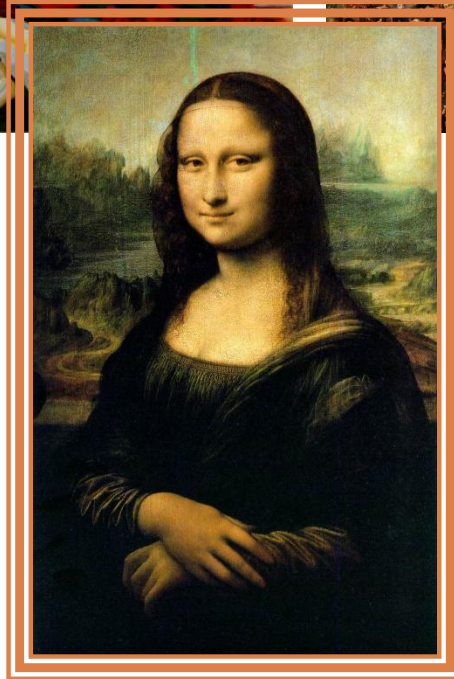
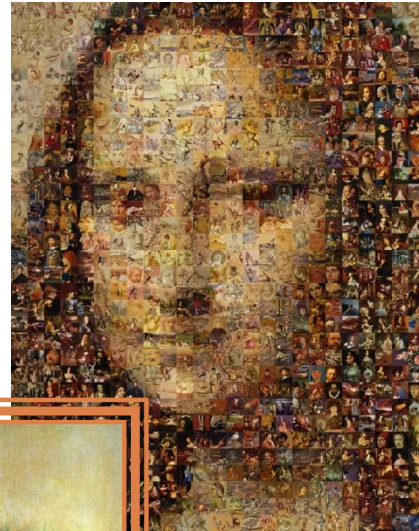
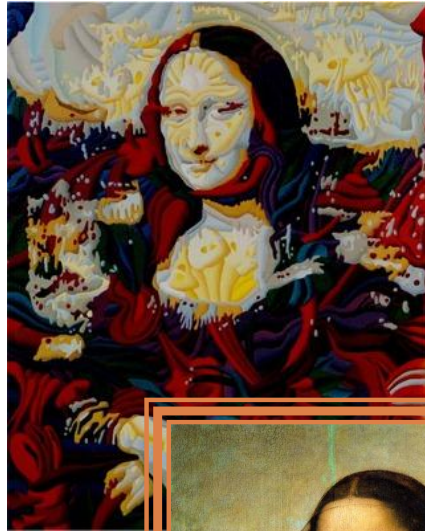
$$G_1 = 0.51\text{PMDCI} + 0.63\text{THIOL} + \dots + 0.26\text{LPIN} - 0.27\text{LPIN1} + \dots - 0.30\text{CYP4A14}$$

	PMDCI	THIOL	CYP3A11	CYP4A10	Lpin	Lpin1	GSTmu	GSTpi2	S14	CYP4A14
PMDCI	1.0000000	0.84318213	0.78800310	0.85333860	-0.15426020	-0.12132735	0.42444476	0.4422014	0.09117950	0.7509967
THIOL	0.8431821	1.0000000	0.73554530	0.79996380	0.07321187	0.09424437	0.56878570	0.3596621	0.33377479	0.6987517
CYP3A11	0.7880031	0.73554530	1.0000000	0.75508008	-0.01894254	-0.03396576	0.62005000	0.6041323	-0.03192741	0.7368760
CYP4A10	0.8533386	0.79996380	0.75508008	1.0000000	-0.22755111	-0.19530806	0.43563040	0.4159180	0.07638834	0.8853495
Lpin	-0.1542602	0.07321187	-0.01894254	-0.22755111	1.00000000	0.97166955	0.10963198	-0.1489248	0.57820562	-0.2886763
Lpin1	-0.1213273	0.09424437	-0.03396576	-0.19530806	0.97166955	1.00000000	0.06449885	-0.1234486	0.58591046	-0.2782659
GSTmu	0.4244448	0.56878570	0.62005000	0.43563040	0.10963198	0.06449885	1.00000000	0.4505130	0.08559840	0.5285176
GSTpi2	0.4422014	0.35966214	0.60413234	0.41591801	-0.14892482	-0.12344856	0.45051300	1.0000000	-0.27370381	0.4171278
S14	0.0911795	0.33377479	-0.03192741	0.07638834	0.57820562	0.58591046	0.08559840	-0.2737038	1.00000000	-0.1123210
CYP4A14	0.7509967	0.69875167	0.73687596	0.88534950	-0.28867628	-0.27826594	0.52851762	0.4171278	-0.11232101	1.0000000

> █

$$r(\text{LPIN}, \text{LPIN1}) = 0.97$$

# Looking for factors common to SEVERAL sets of variables



# Generalized Canonical Analysis

$X_1$

$X_2$

...

$X_J$

# Generalized Canonical Analysis

$X_1$

$X_2$

...

$X_J$

$$z_s / \sum_j R^2(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$

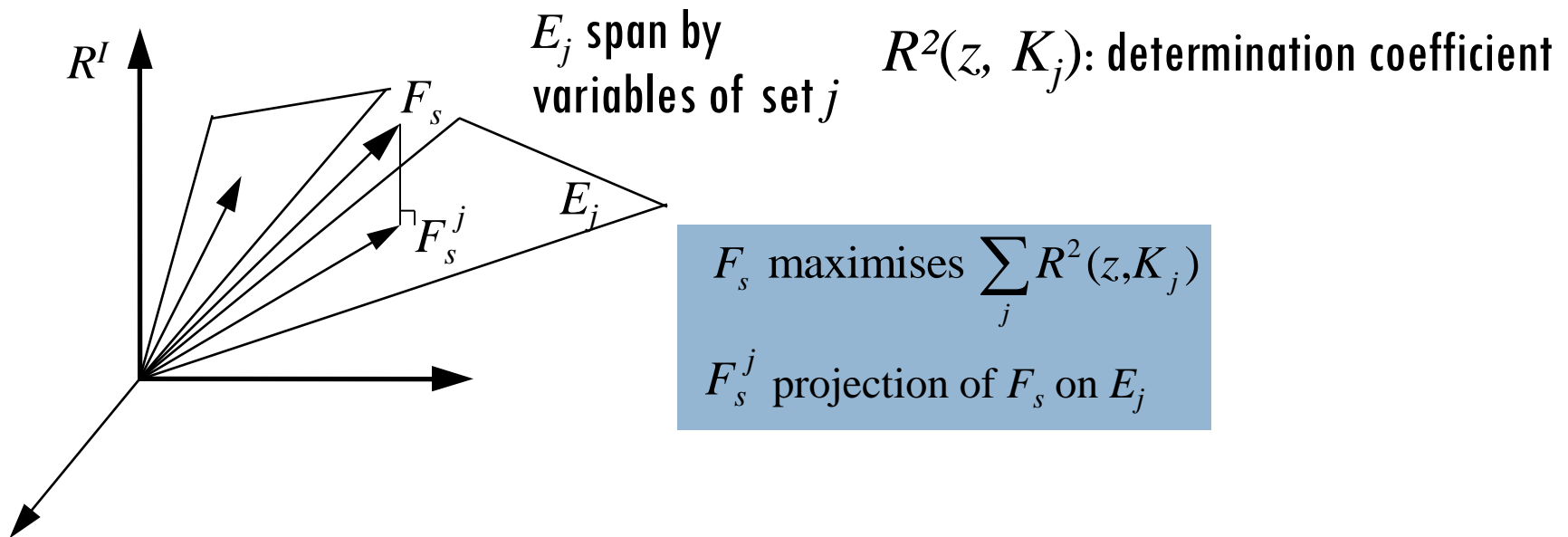
# Generalized Canonical Analysis

For each step  $s$ :

Firstly: general variable (related to all the sets of variables)

Secondly: canonical variables

(linear combination of variables of sets  $j$  related to general variable)



Generalized canonical analysis (Carroll, 1968)

# Generalized Canonical Analysis

$K_1$

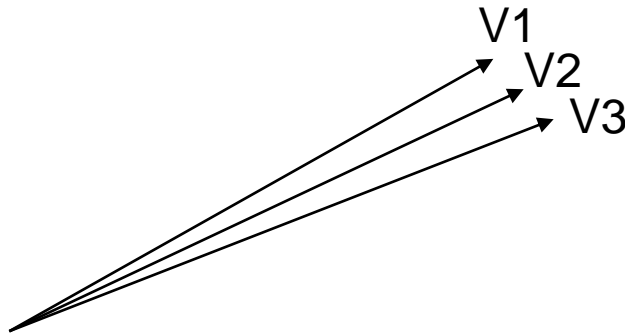
$K_2$

...

$K_J$

$$z_s / \sum_j R^2(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$



# Generalized Canonical Analysis

$K_1$

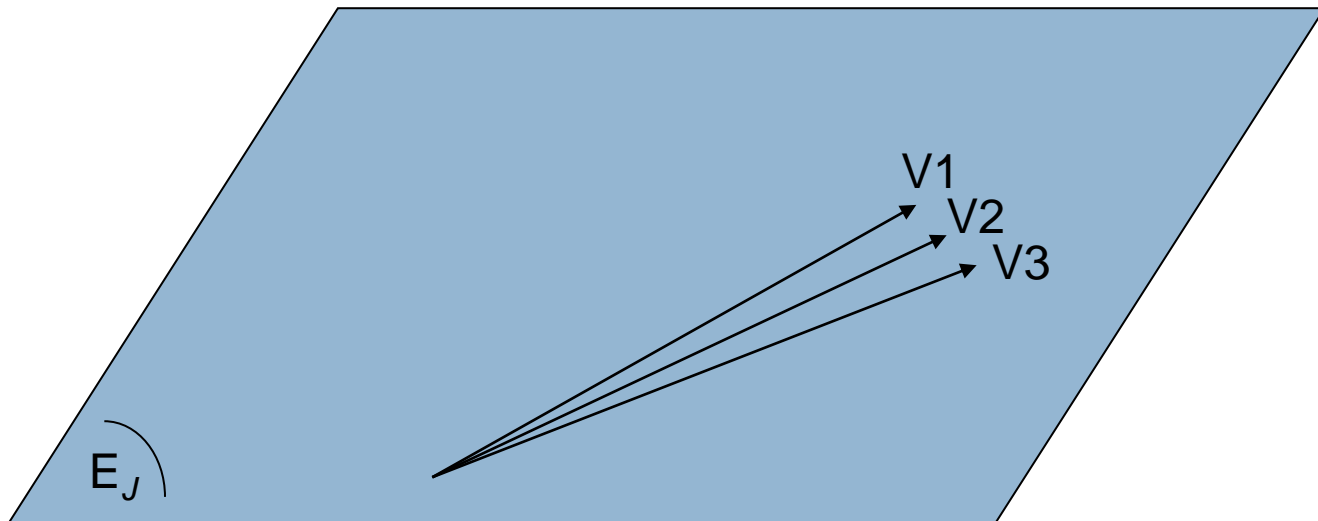
$K_2$

...

$K_J$

$$z_s / \sum_j R^2(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$



# Generalized Canonical Analysis

$K_1$

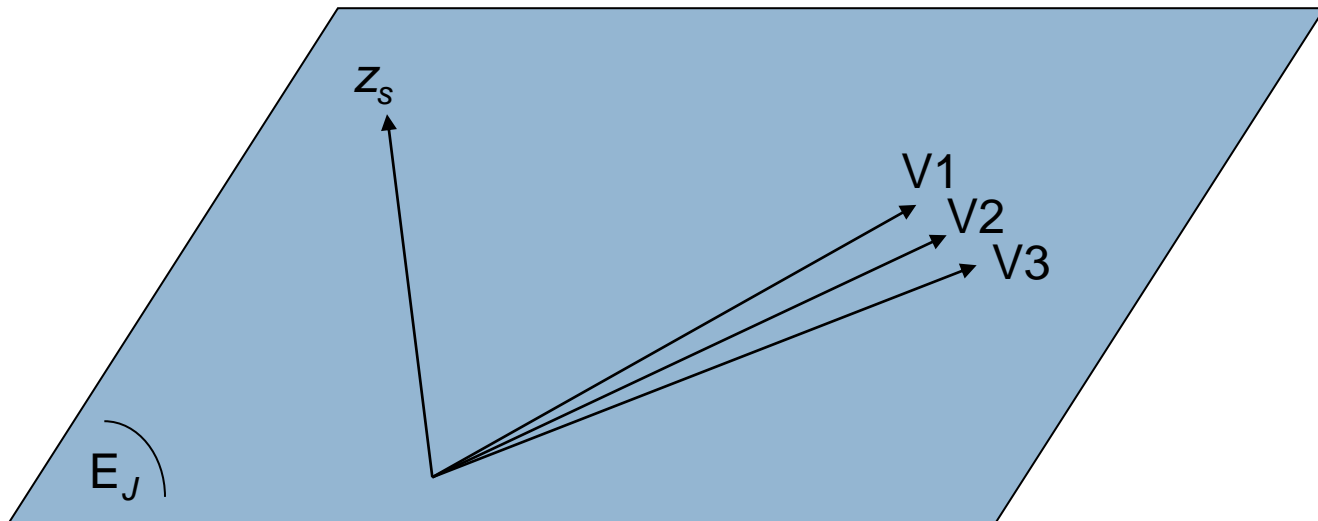
$K_2$

...

$K_J$

$$z_s / \sum_j R^2(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$



# Generalized Canonical Analysis

$K_1$

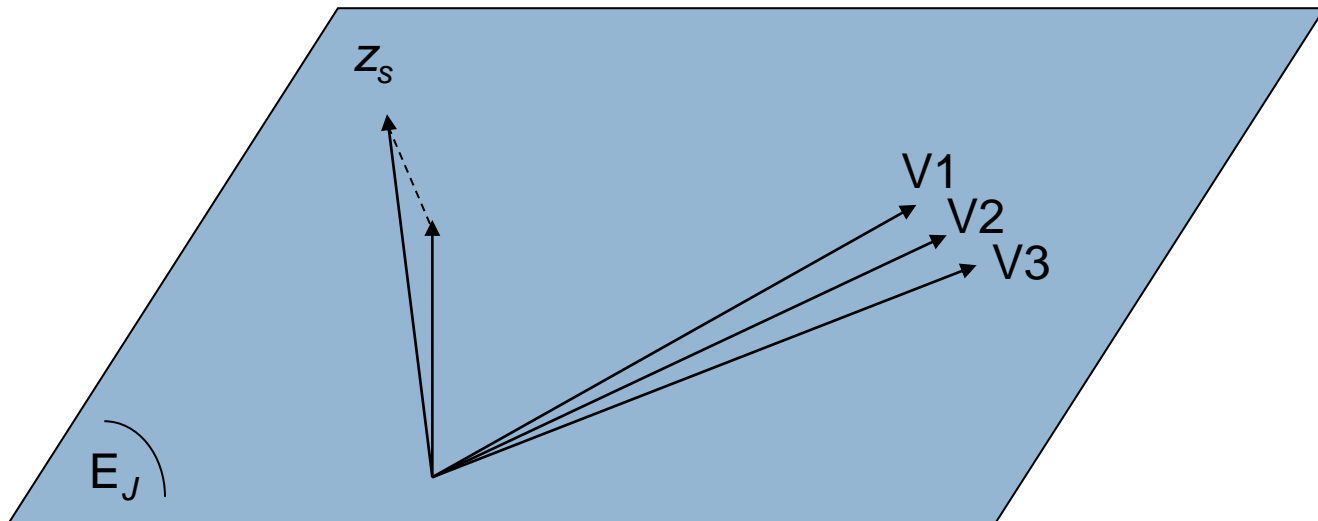
$K_2$

...

$K_J$

$$z_s / \sum_j R^2(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$



# Multiple Factor Analysis

$X_1$

$X_2$

...

$X_J$

# Multiple Factor Analysis

$X_1$

$X_2$

...

$X_J$

$$z_s / \sum_j Lg(z_s, K_j) \text{ is max}$$

$$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$$

# Multiple Factor Analysis

A measure of relationship between

one variable  $z$

a set of variables  $K_j = \{v_k ; k = 1, K_j\}$

$Lg(z, K_j) =$  projected inertia of the whole set of the variables  $v_k$  onto  $z$

Case of standardized variables weighted in a MFA

$$Lg(z, K_j) = \frac{1}{\lambda_1^j} \sum_{k \in K_j} r^2(z, v_k)$$

In every case, owing to the weighting of MFA:

$$0 \leq Lg(z, K_j) \leq 1$$

# Multiple Factor Analysis

$K_1$

$K_2$

...

$K_J$

$z_s / \sum_j \text{Inertia of } K_j \text{ projected on } z_s$  is max

$\text{Var}(z_s) = 1$  and  $\text{Cor}(z_s, z_t) = 0, \forall t < s$

$Lg(z, K_j) = \text{Inertia of } K_j \text{ projected on } z$

$$= \frac{1}{\lambda_1^j} \sum_{k \in K_j} \text{Cor}^2(z, k)$$

$Lg(z, K_j) = 1 \Leftrightarrow z$  is the 1<sup>st</sup> princ. comp. of  $K_j$

# Multiple Factor Analysis

$K_1$

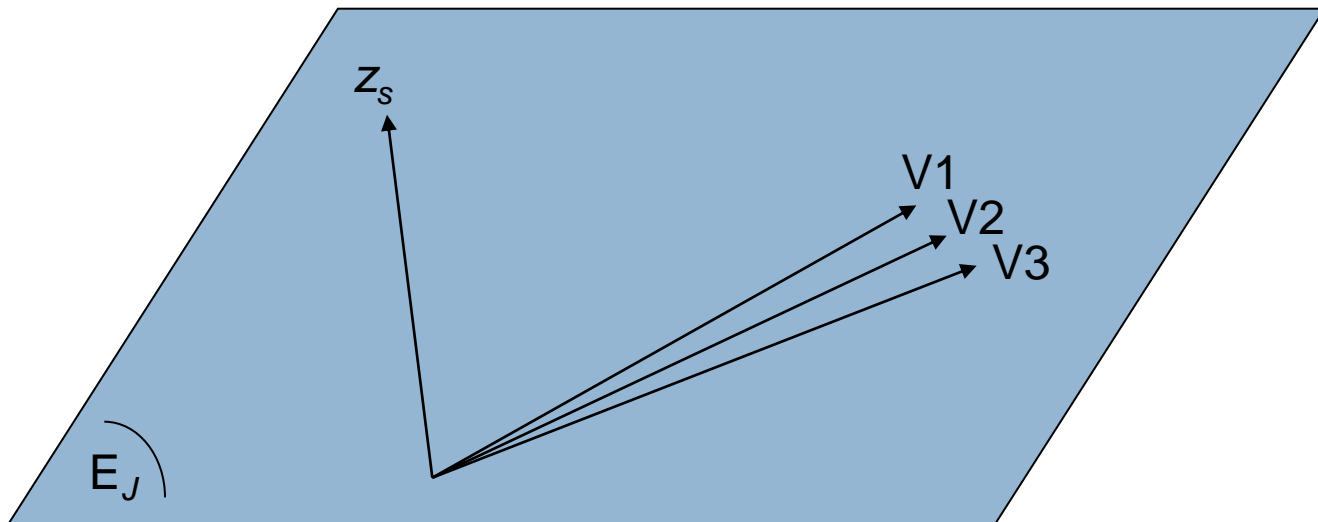
$K_2$

...

$K_J$

$z_s / \sum_j \text{Inertia of } K_j \text{ projected on } z_s \text{ is max}$

$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$



# Multiple Factor Analysis

$K_1$

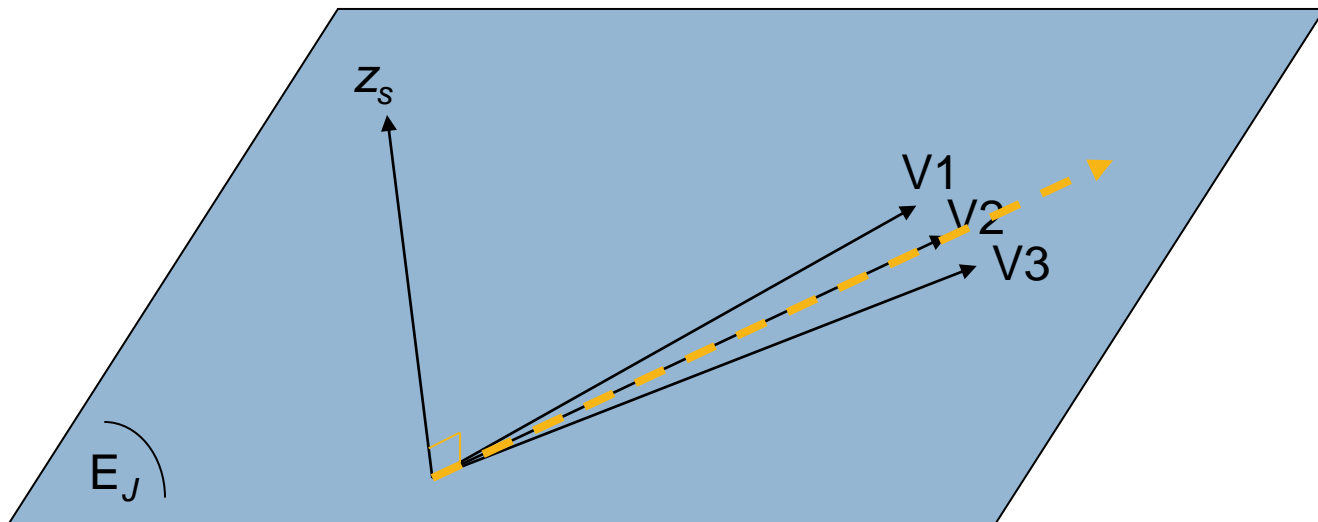
$K_2$

...

$K_J$

$z_s / \sum_j \text{Inertia of } K_j \text{ projected on } z_s \text{ is max}$

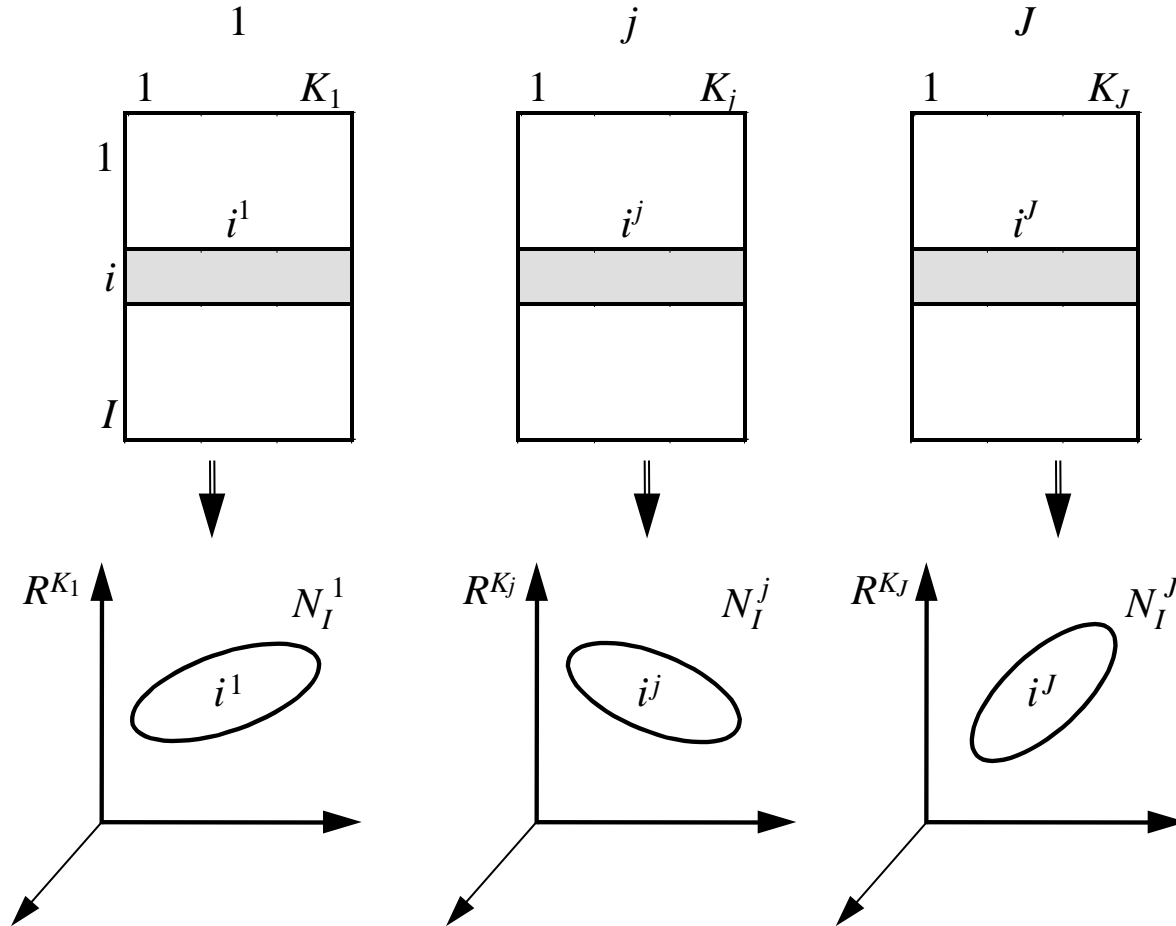
$\text{Var}(z_s) = 1 \text{ and } \text{Cor}(z_s, z_t) = 0, \forall t < s$



SUPERIMPOSED  
REPRESENTATION OF THE  $J$   
CLOUDS OF INDIVIDUALS

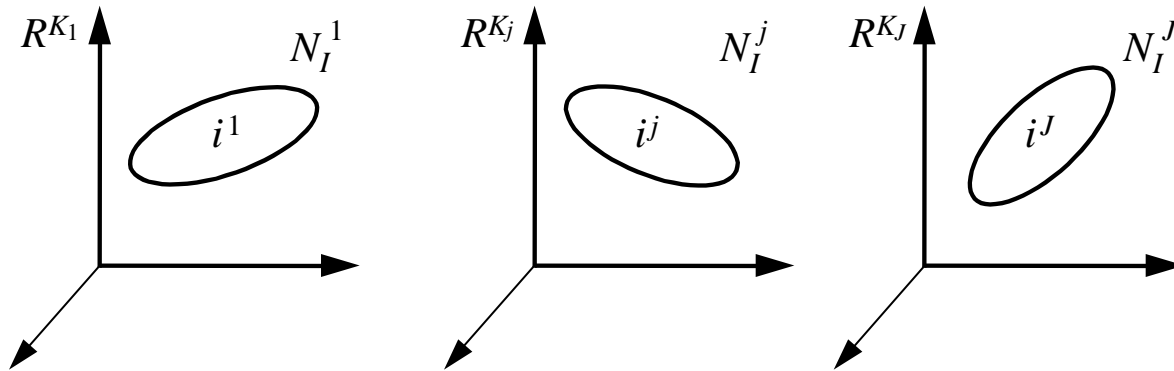


# Superimposed representation of the $J$ clouds of individuals



$N_I^j$  : partial cloud (of individuals; relatively to the set  $j$ )

# Superimposed representation of the $J$ clouds of individuals



How to compare clouds representing the same objects but in different spaces ?

Reference method: Procrustes analysis (Green, 1952; Gower, 1975)

# Superimposed representation of the *J* clouds of individuals

- Procrustes was a character of Greek myth. An innkeeper who plied his trade in Attica, he put his victims on an iron bed. If they were longer than the bed, he cut off their feet. If they were shorter, he stretched them...

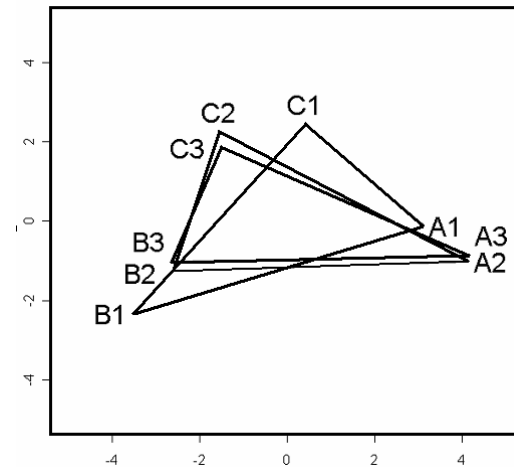
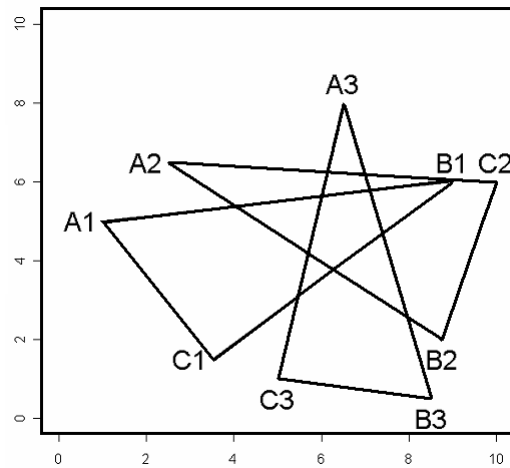
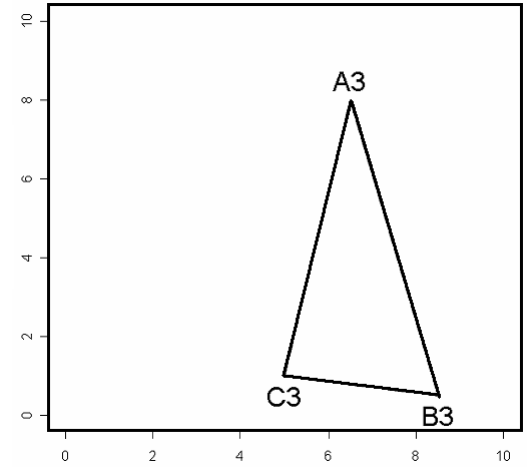
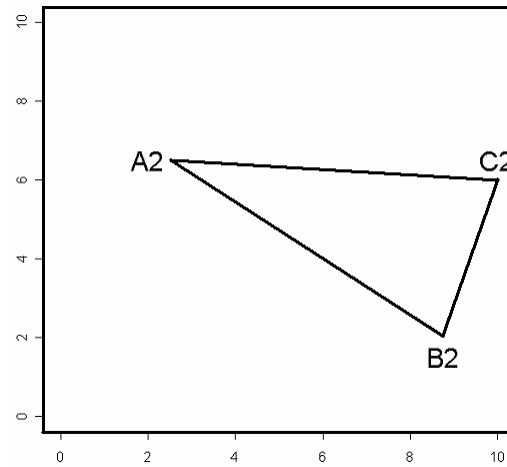
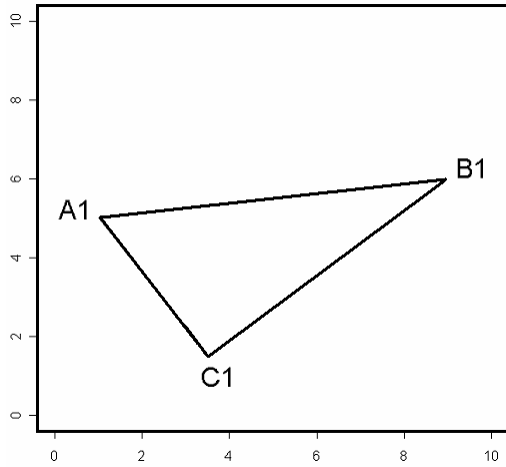


# Superimposed representation of the $J$ clouds of individuals

Make the configurations fit each other

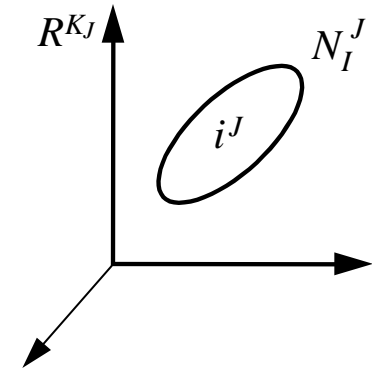
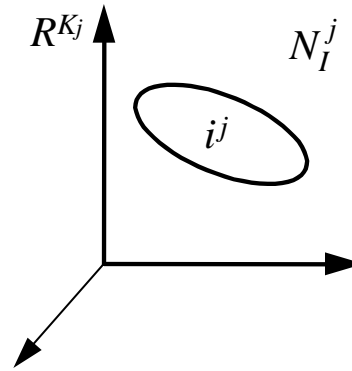
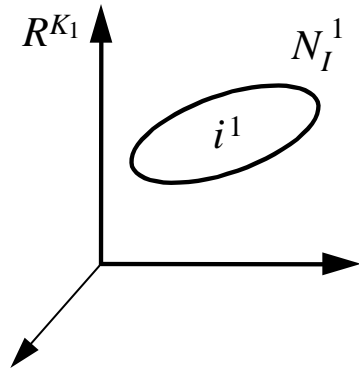
- do this by moving them to a common origin
- stretch or shrink each configuration in order to make it fit as good as possible
- if needed, flip them around

# Superimposed representation of the $J$ clouds of individuals



# Superimposed representation of the $J$ clouds of individuals

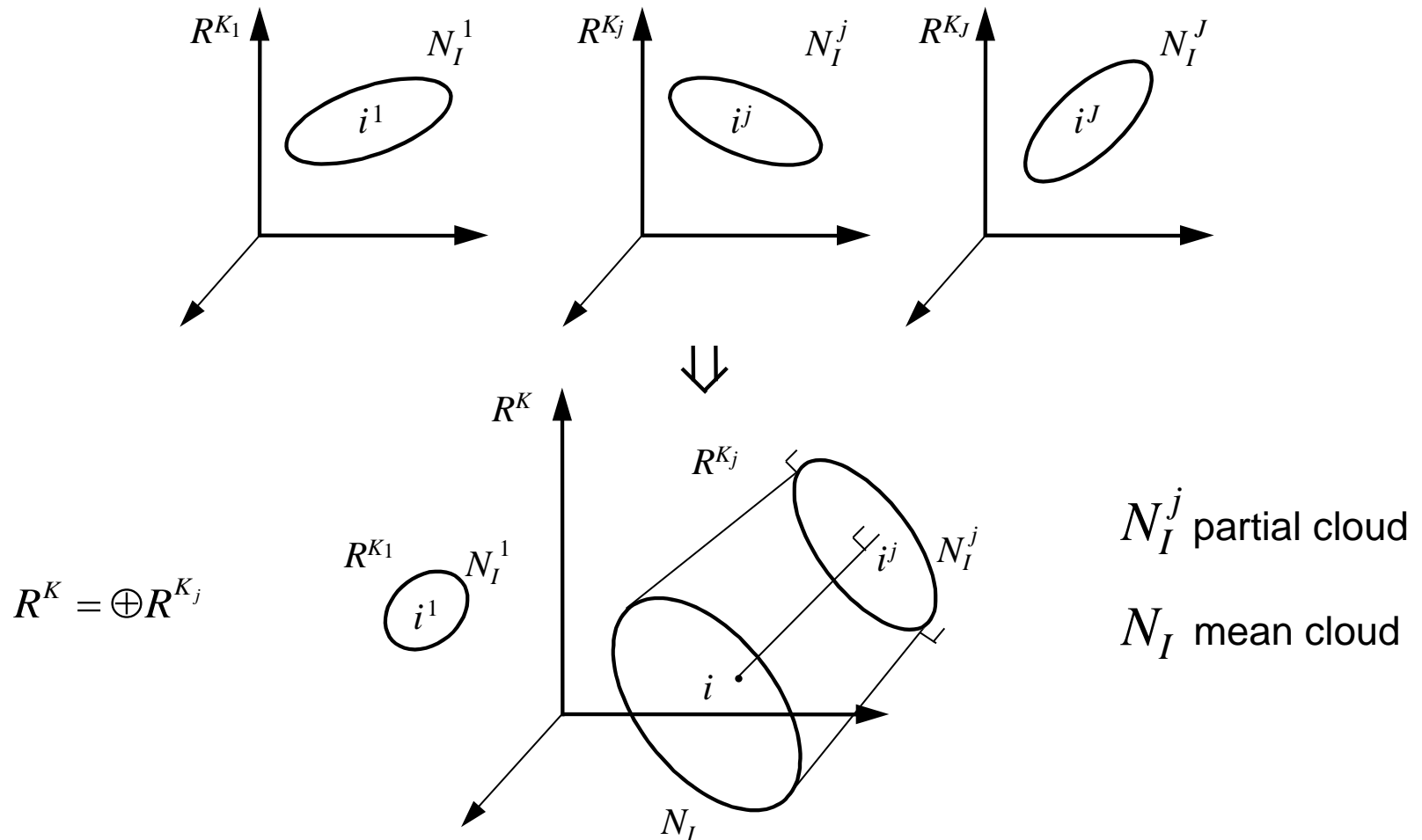
## Geometrical framework



- 1)  $N_I^j$  must be well represented
- 2) The  $J$  points representing the same individual must be close to one another

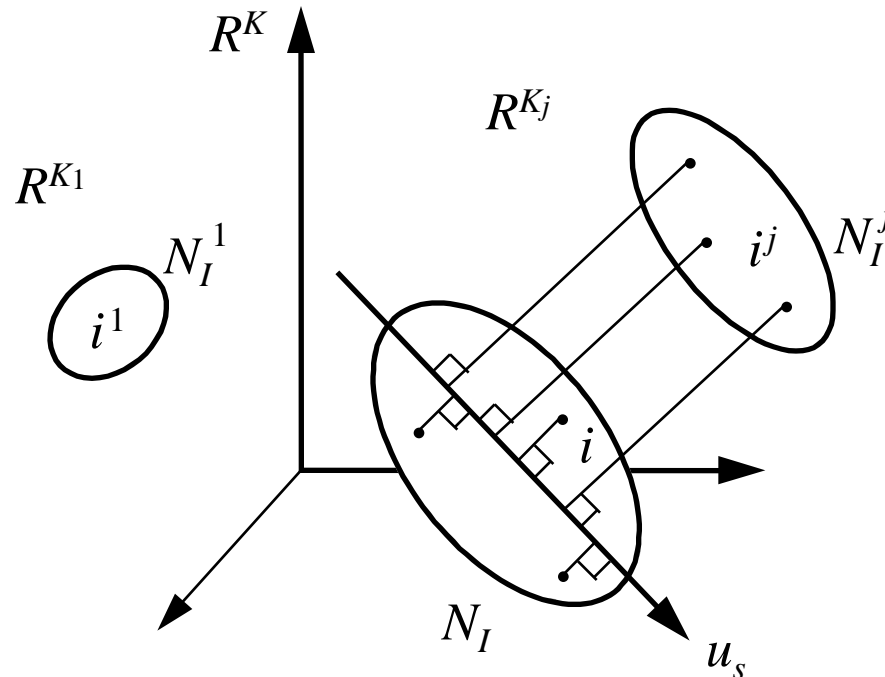
# Superimposed representation of the $J$ clouds of individuals

## Geometrical framework



# Superimposed representation of the $J$ clouds of individuals

## Principle



The partial clouds are projected onto the principal components of the mean cloud

# Superimposed representation of the $J$ clouds of individuals

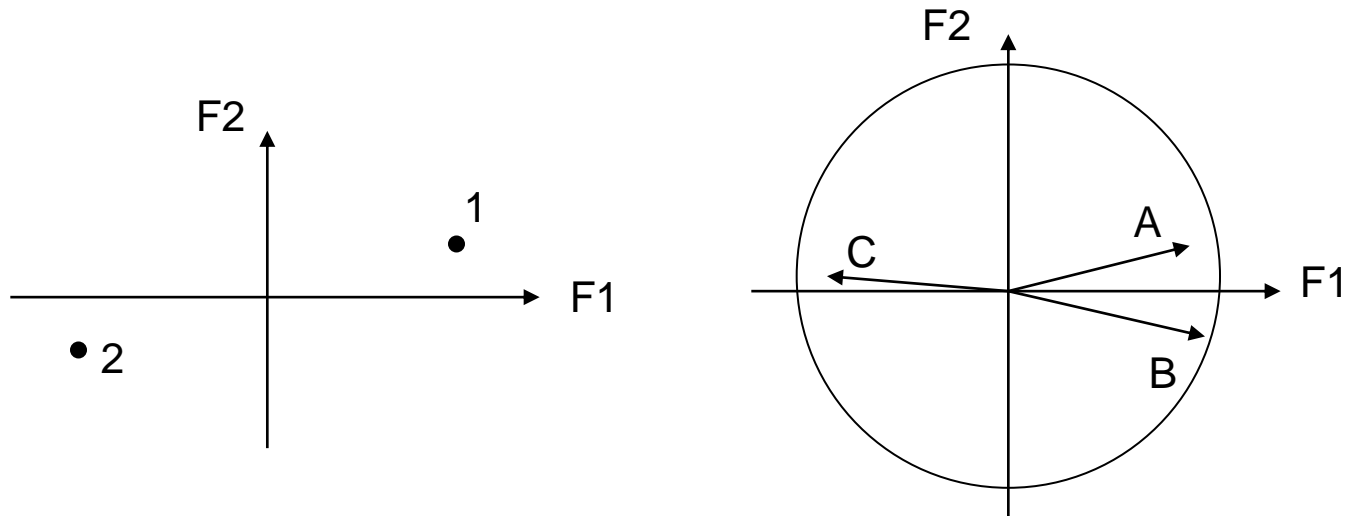
- The superimposed representation and the canonical variables provided by MFA express a very same problematic since they both correspond to the same solution to two apparently different problems

# Superimposed representation of the $J$ clouds of individuals

## Usual transition relationship in PCA

$$F_s(i) = \frac{1}{\sqrt{\lambda_s}} \sum_{k \in K} x_{ik} G_s(k)$$

- $F_s(i)$  coordinate of  $i$  along the axis  $s$
- $G_s(k)$  coordinate of variable  $k$  along the axis  $s$
- $\lambda_s$  eigenvalue associated to the axis  $s$
- $x_{ik}$  data (value of  $k$  for  $i$ )



# Superimposed representation of the $J$ clouds of individuals

Usual transition relationship in PCA

$$F_s(i) = \frac{1}{\sqrt{\lambda_s}} \sum_{k \in K} x_{ik} G_s(k)$$

If the variable  $k$  has the weight  $m_k$

$$F_s(i) = \frac{1}{\sqrt{\lambda_s}} \sum_{k \in K} x_{ik} m_k G_s(k)$$

Usual transition relationship applied to the mean cloud in MFA

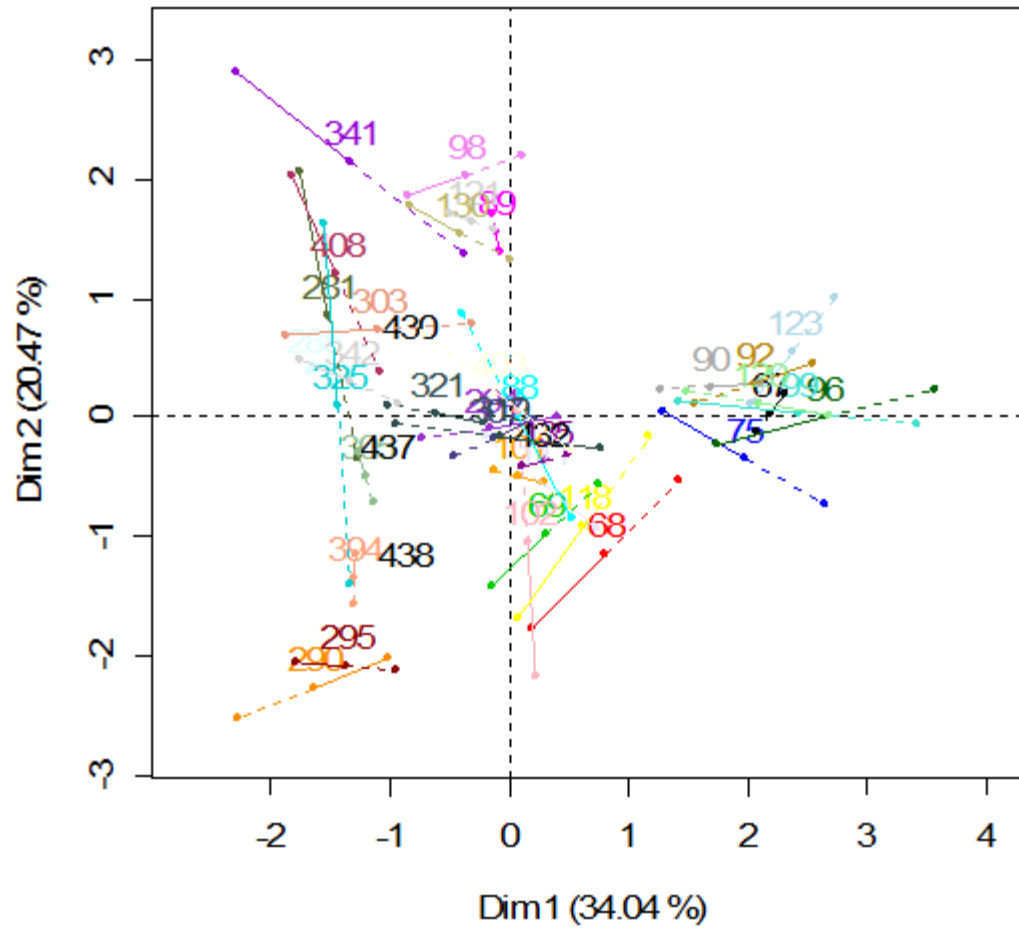
$$F_s(i) = \frac{1}{\sqrt{\lambda_s}} \sum_{j \in J} \frac{1}{\lambda_1^j} \sum_{k \in K_j} x_{ik} G_s(k)$$

Partial transition relationship

$$F_s(i^j) = \frac{1}{\sqrt{\lambda_s}} \frac{J}{\lambda_1^j} \sum_{k \in K_j} x_{ik} G_s(k)$$

$$F_s(i) = \frac{1}{J} \sum_{j \in J} F_s(i^j)$$

Individual factor map

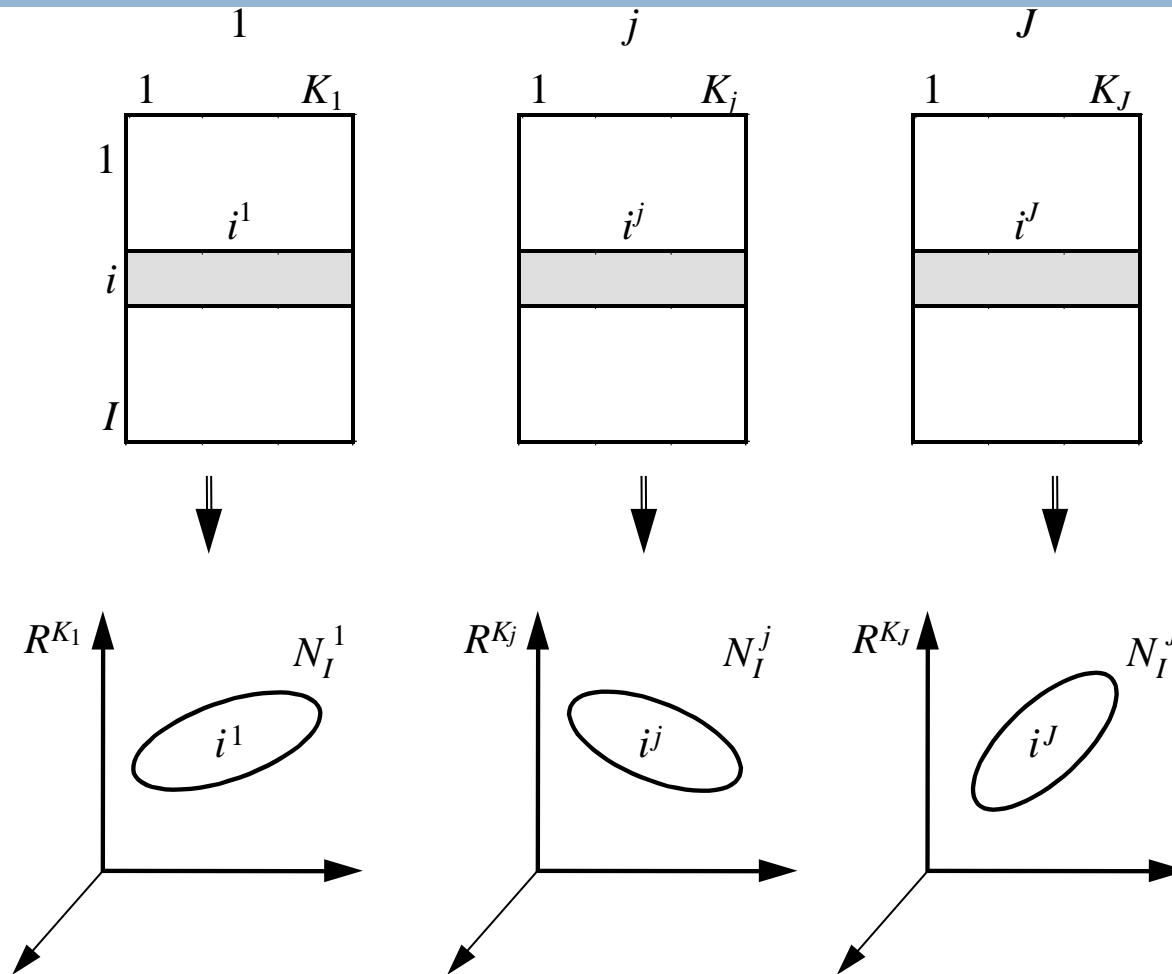




# GLOBAL REPRESENTATION OF SETS OF VARIABLES



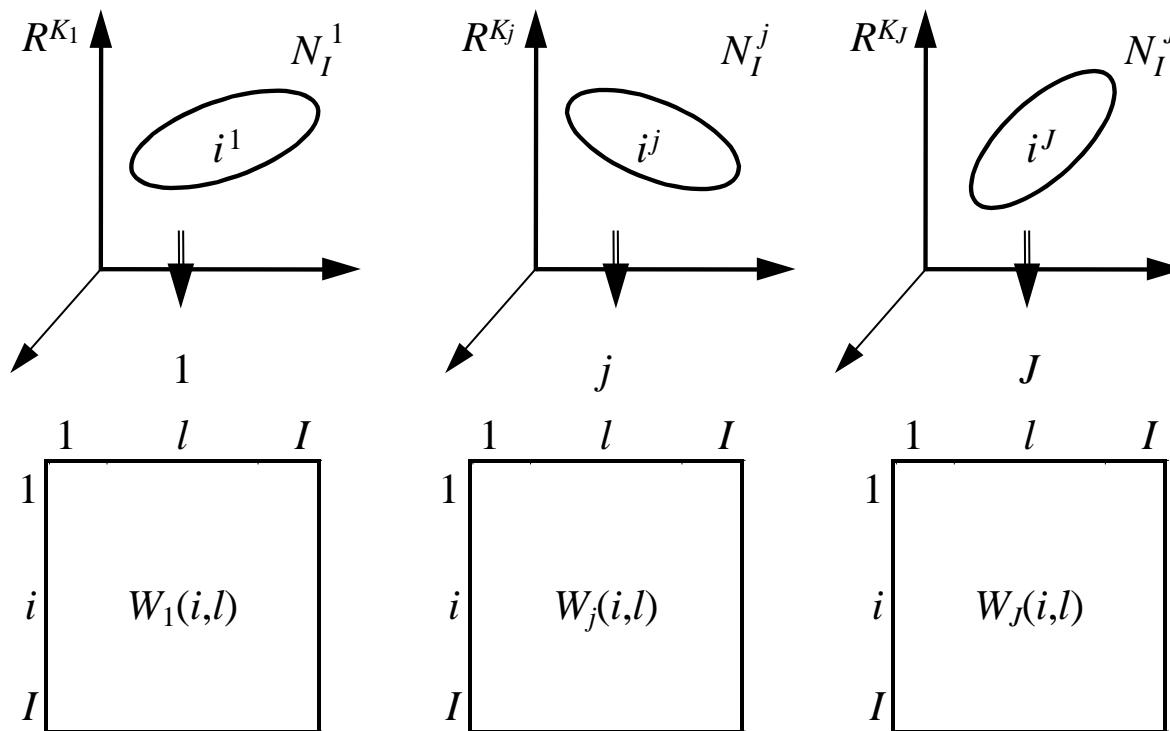
# Global representation of sets of variables



$N_I^j$  : partial cloud (of individuals ; associated to the set  $j$ )

# Global representation of sets of variables

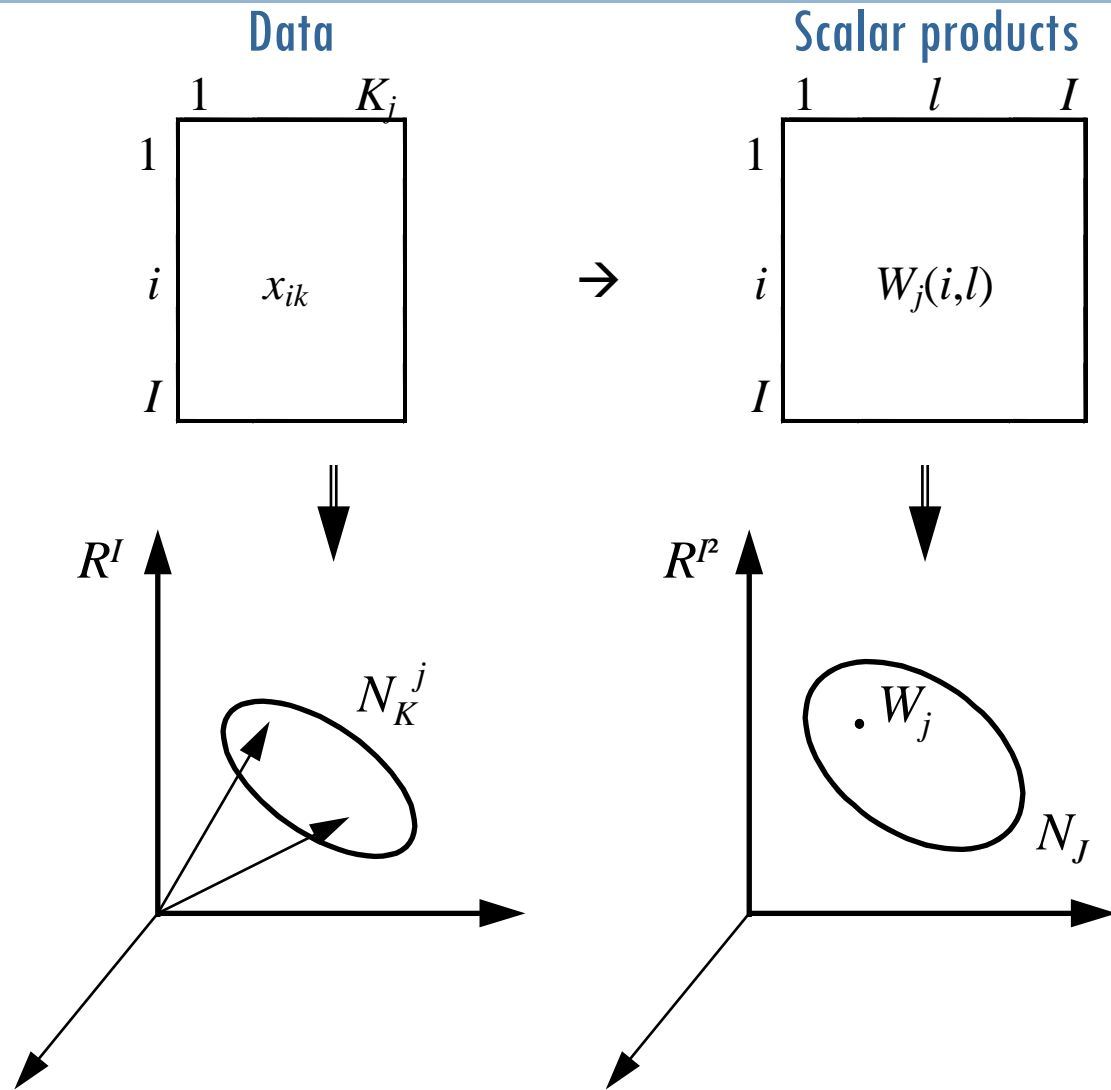
How to measure the global resemblance of the  $N_I^j$  ?



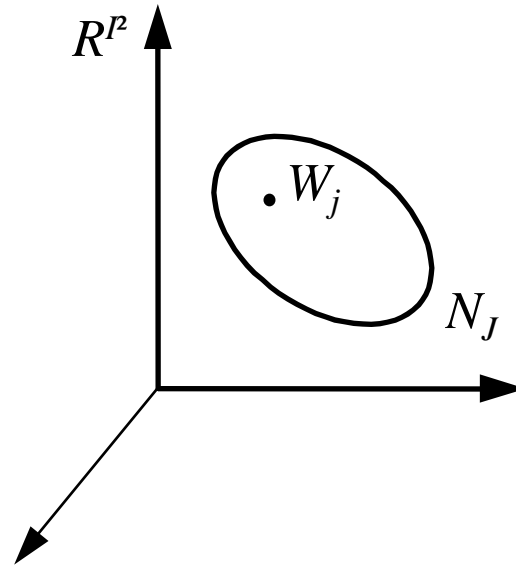
Matrices of scalar products between individuals for each set of variables

$$W_j = X_j X_j'$$

# Global representation of sets of variables



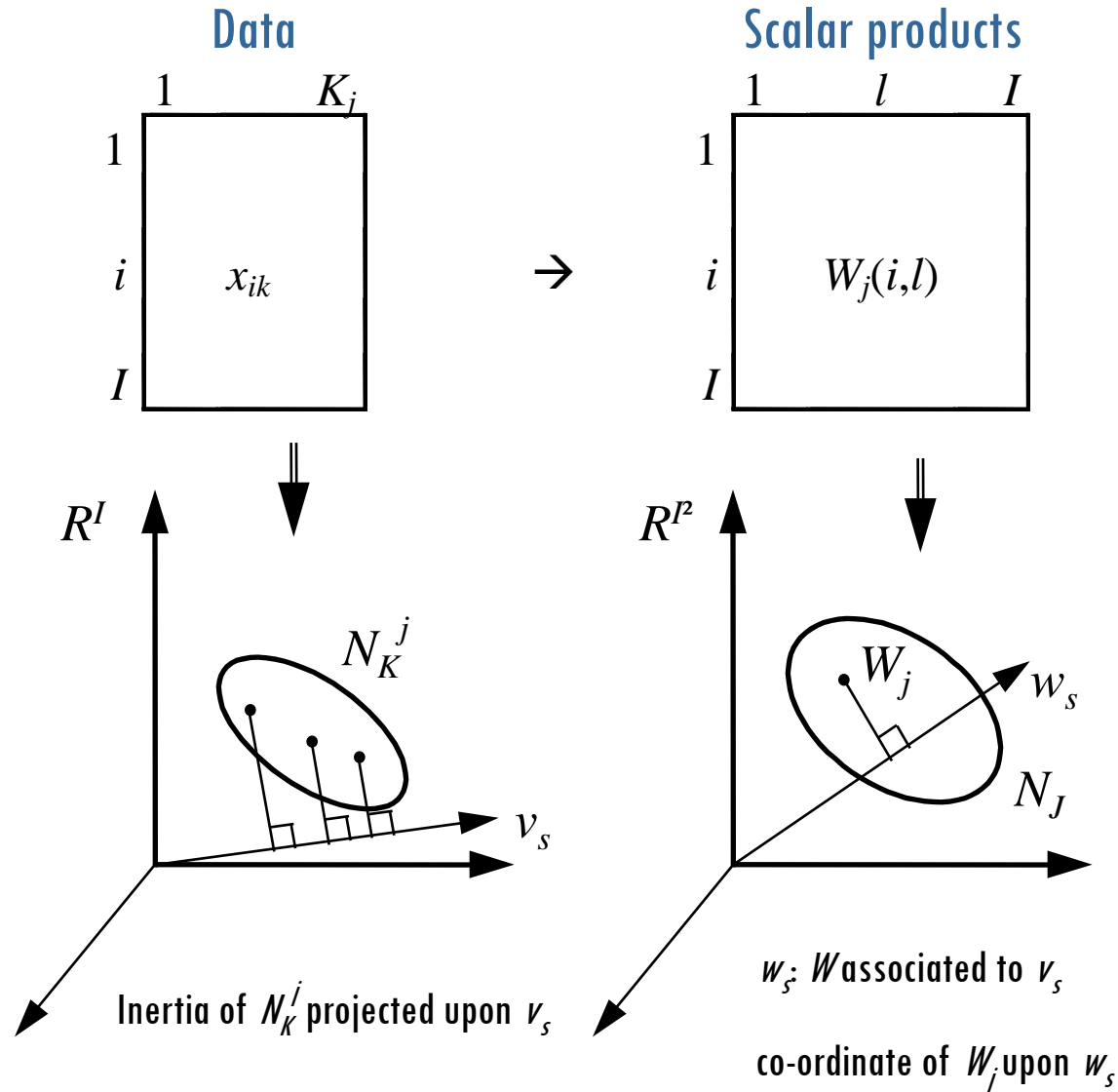
# Global representation of sets of variables



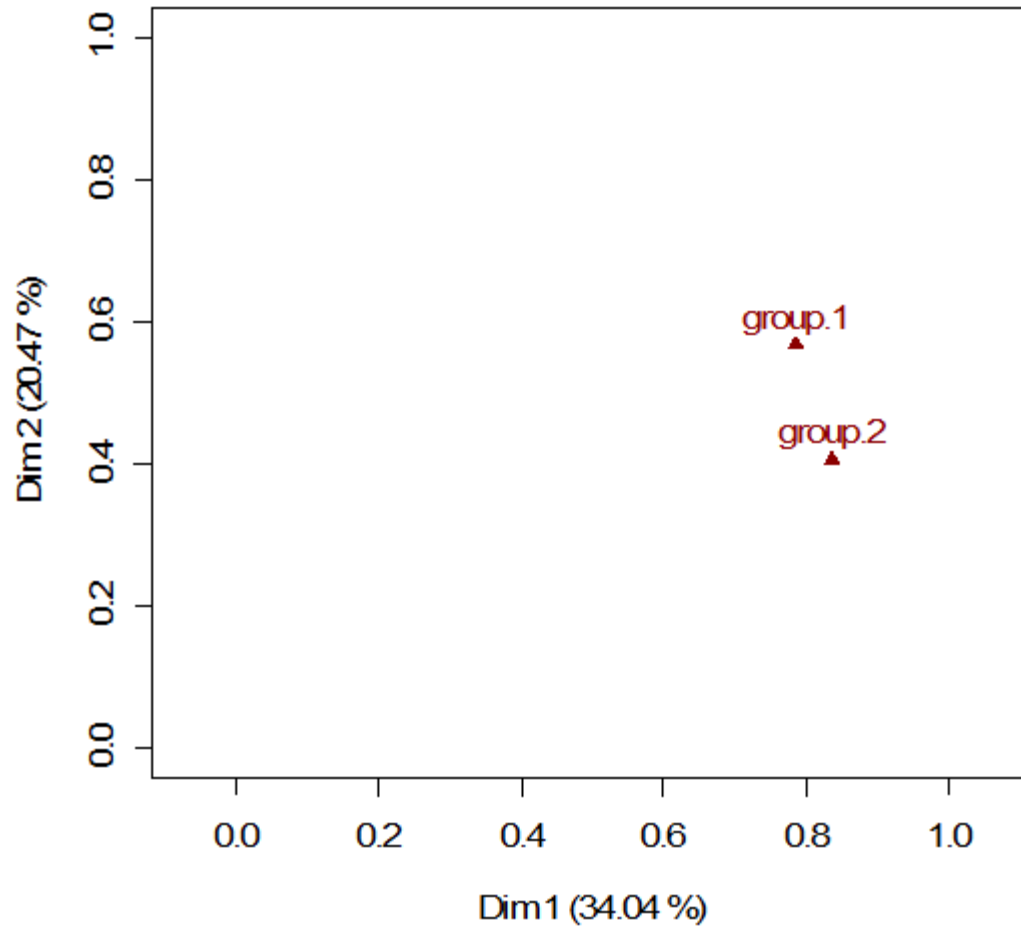
Studying the cloud  $N_j$

Reference method: STATIS (Escoufier Y., Lavit C.)

# Global representation of sets of variables



### Groups representation



“THANK YOU FOR BEING  
HERE”

[http://www.ted.com/talks/john\\_francis\\_walks\\_the\\_earth.html](http://www.ted.com/talks/john_francis_walks_the_earth.html)