Euclidean representations of a set of hierarchies using Multiple Factor Analysis

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Outline

Introduction

Interested in:

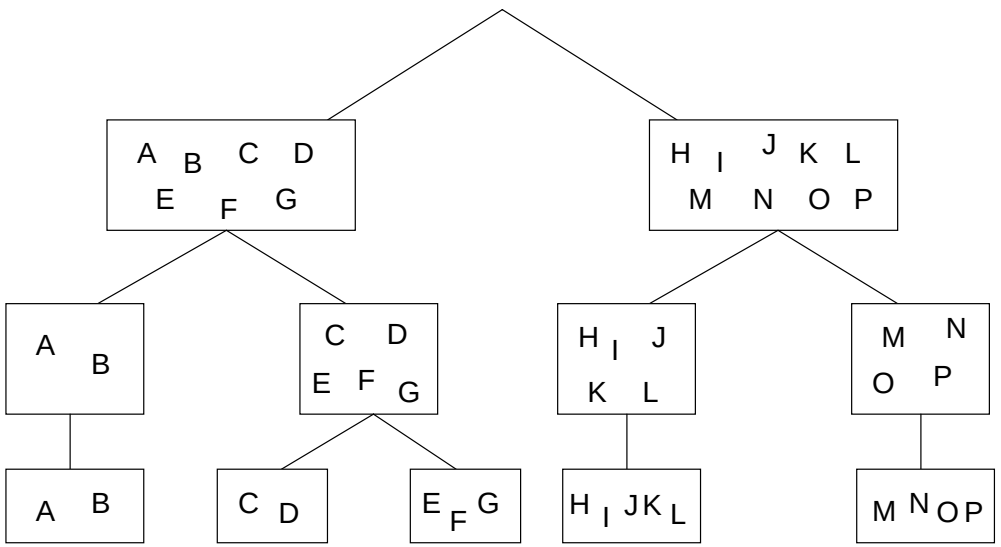
- Set of non-indexed hierarchies
- Synthetic graphical representations

At least 2 possible graphical representations:

- As a hierarchy consensus (Adams, 1972)
  - Same shape of the data
  - Consensus difficult to obtain when the number of hierarchies increases
- As an Euclidean representation of the hierarchies: representation of the terminal nodes, etc.

Outline

Data coding (1)



	L1	L2	L3
A			
B			
C			
D			
E			
F			
G			
H			
I			
J			
K			
L			
M			
N			
O			
P			

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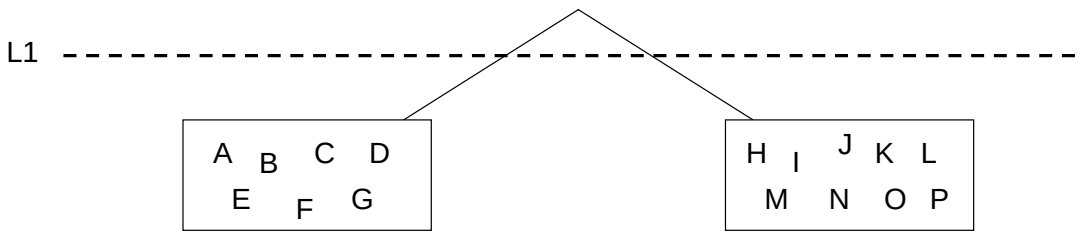

	L1	L2	L3
A	G1		
B	G1		
C	G1		
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E	G1		
F	G1		
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H	G2		
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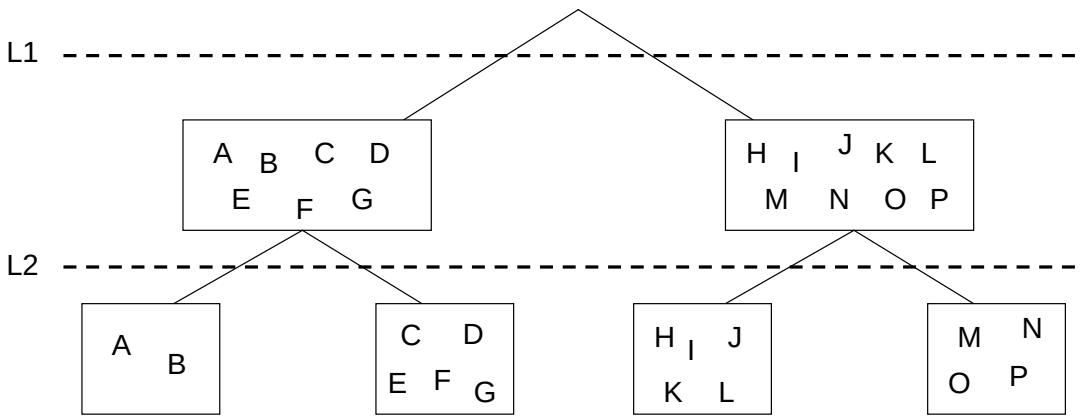

	L1	L2	L3
A	G1	G1	
B	G1	G1	
C	G1	G2	
D	G1	G2	
E	G1	G2	
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M	G2	G4	
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P	G2	G4	

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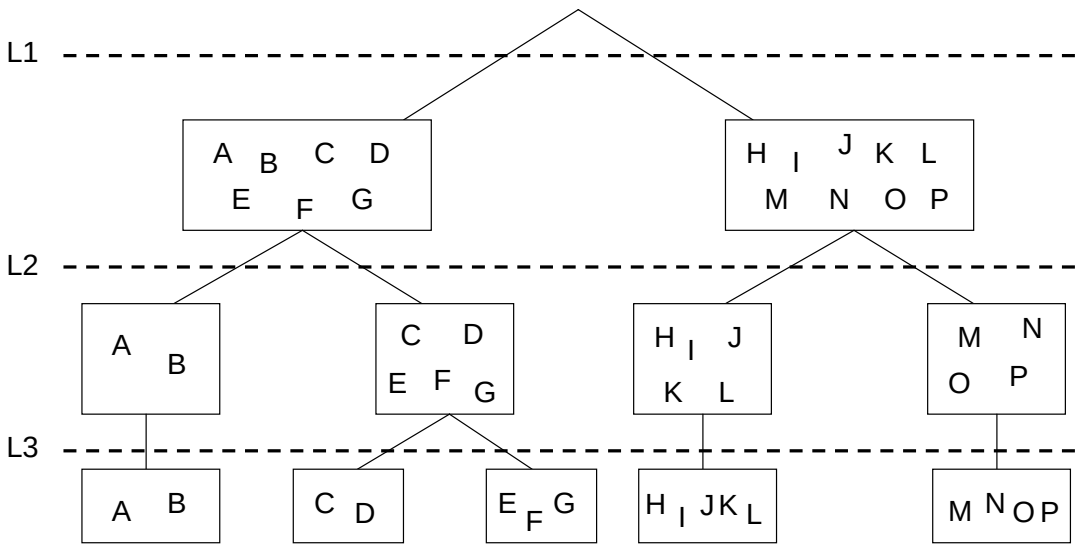

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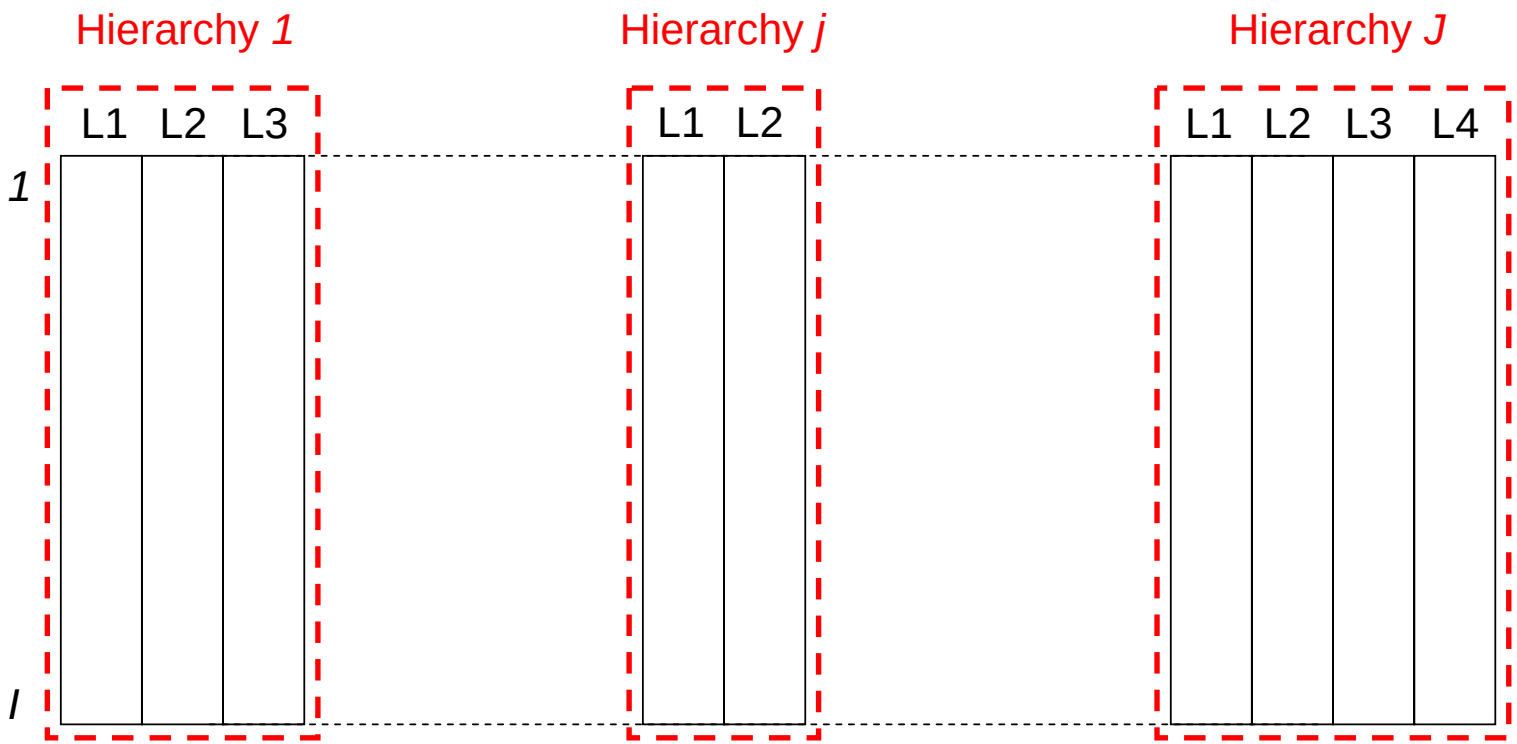
I	G2	G3	G4
J	G2	G3	G4
K	G2	G3	G4
L	G2	G3	G4
M	G2	G4	G5
N	G2	G4	G5
O	G2	G4	G5
P	G2	G4	G5

Data coding (2)



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- Data table with qualitative variables
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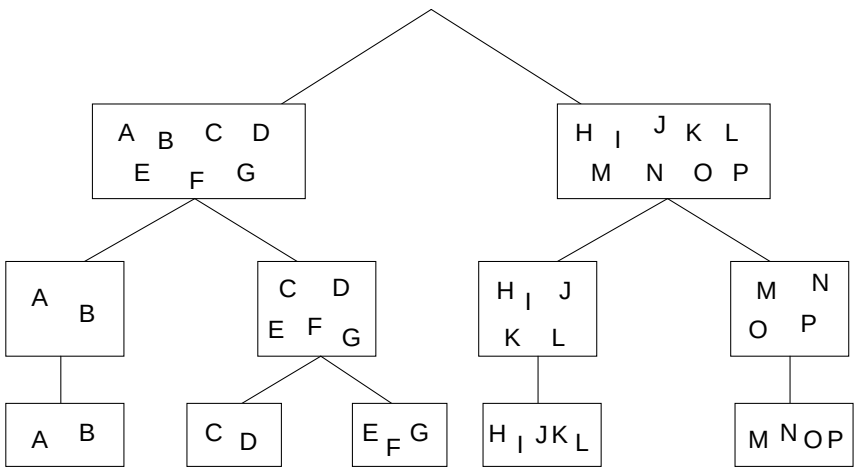

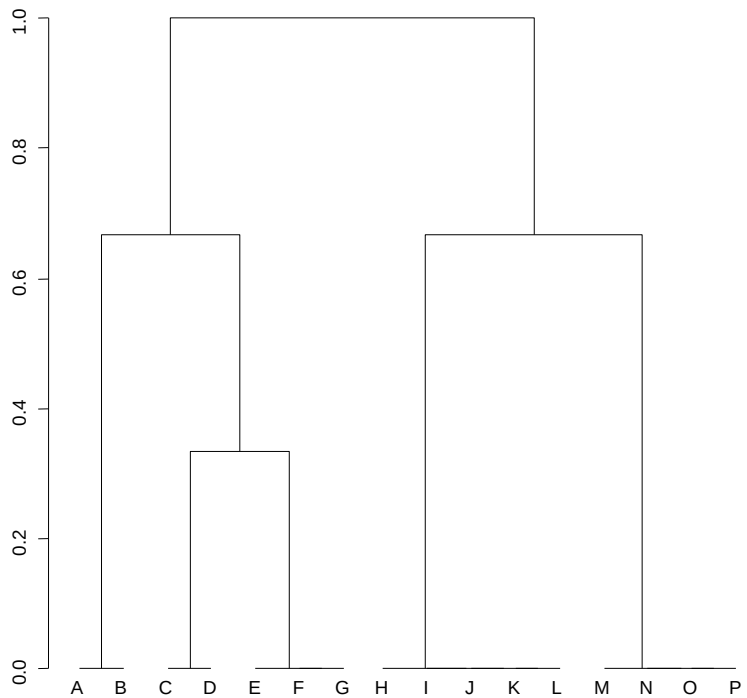

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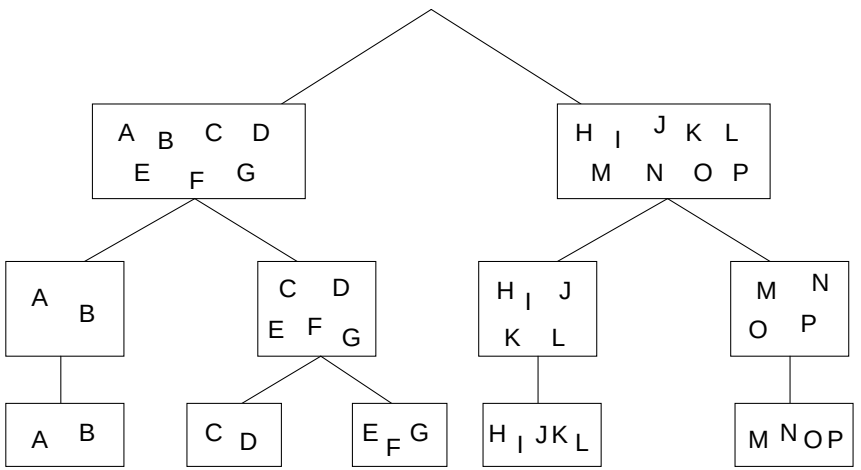

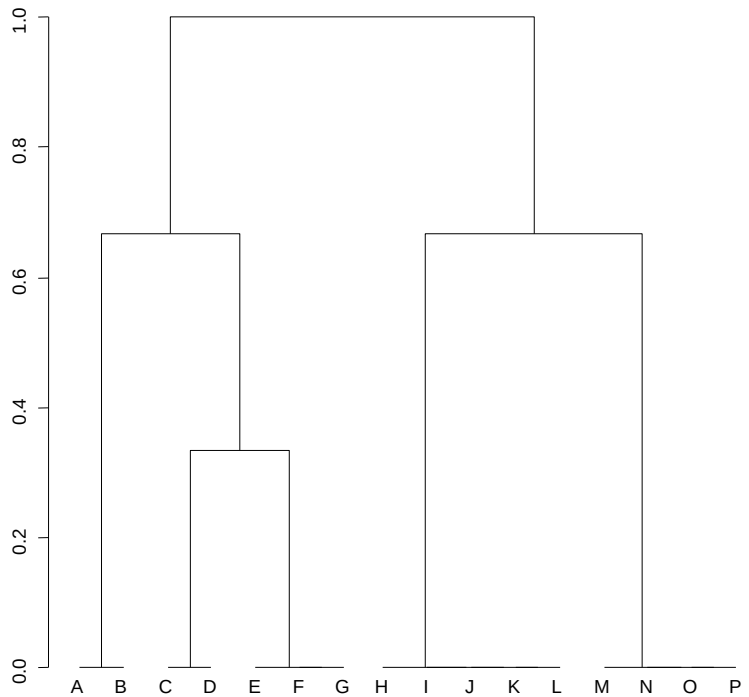

$\Rightarrow$ We found the initial hierarchy

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From a data table with a group structure on the variables, we want to perform a global factorial analysis such as:

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$\Rightarrow$ Multiple Factor Analysis (MFA; Escofier and Pagès (1982)) in which 1 hierarchy corresponds to 1 group of variables

Multiple Correspondence Analysis (MCA)

MCA is looking for dimensions z_s that maximize:

$$\frac{1}{Q}\sum_{q}^{Q}\eta^2(z_s, L_q),$$

with:

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- z_s the axis s
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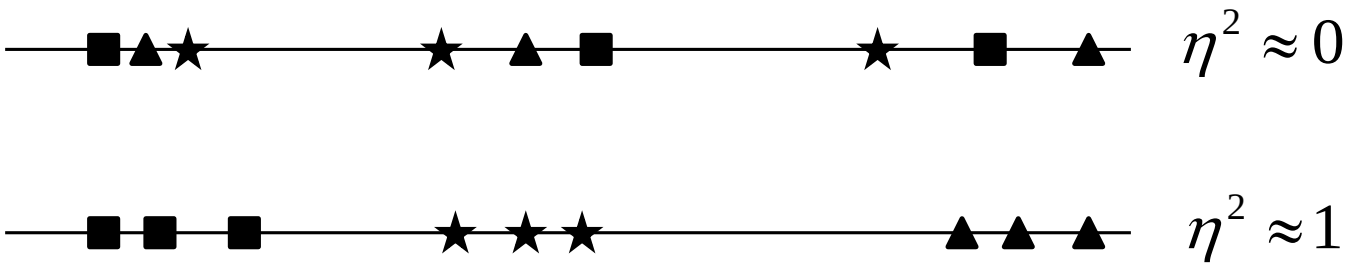

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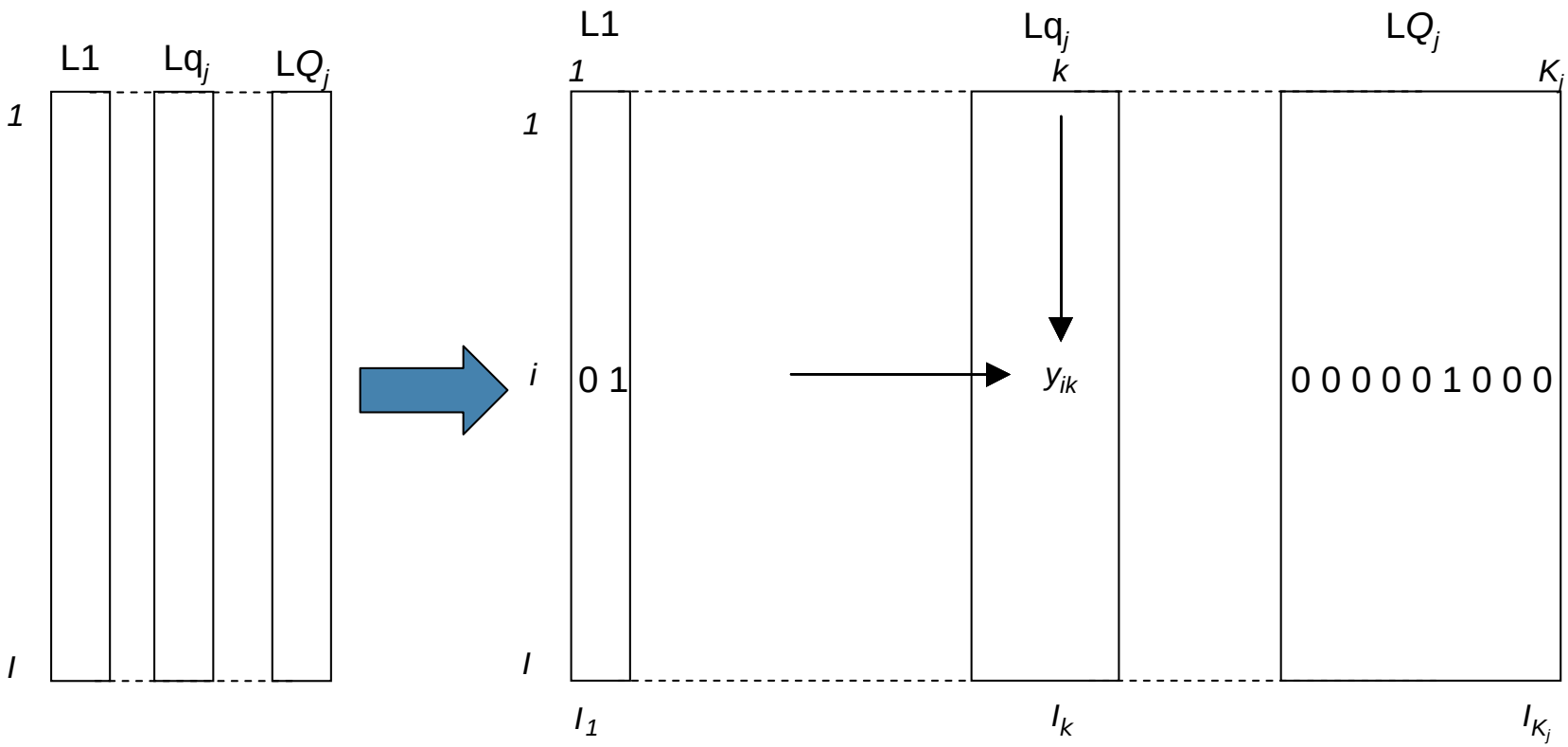
$\Rightarrow$ In this particular case: criterion maximized by MFA $\Leftrightarrow$ sum of criteria maximized by MCA

Disjunctive data table associated with one hierarchy j



Each level (associated with a hierarchy) is represented by a set of dummy variables

Object representation

Distance between 2 objects:

$$d^2(i,l) = \sum_j \frac{1}{Q_j} \sum_{k \in K_j} \frac{I}{I_k}(y_{ik} - y_{lk})^2 = \sum_j d^2_{MCA_j}(i,l),$$

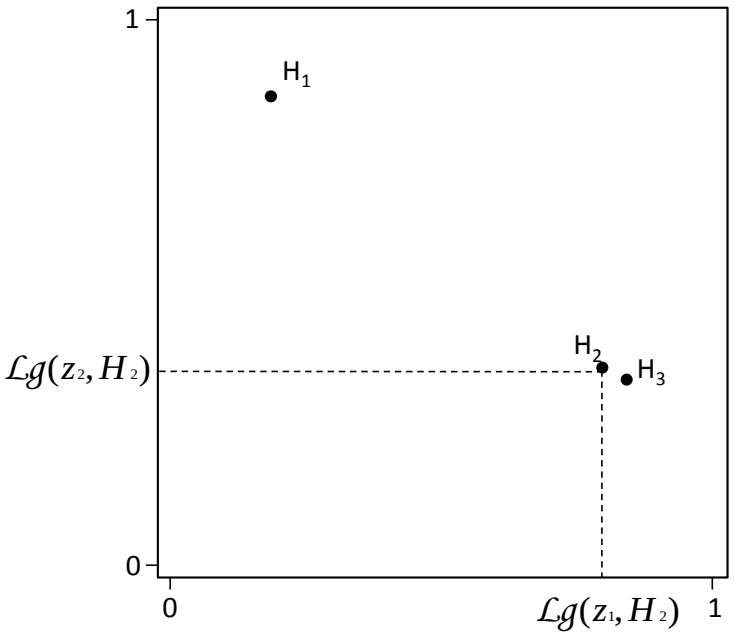
with:

- Q_j the number of level of hierarchy j
- I the number of objects
- I_k the number of objects into the group k
- y_{ik} the element of the disjunctive data table which is equal to 1 if the object i belong to group k and 0 in the opposite case

In this particular case: sum of usual distance in MCA

$\Rightarrow$ 2 objects will be closer than they belong to the same group for a lot of hierarchies

Global hierarchy representation



Coordinate of hierarchy j on axis s:

$$\frac{1}{Q_j} \sum_{q \in Q_j} \eta^2(z_s, L_q),$$

with:

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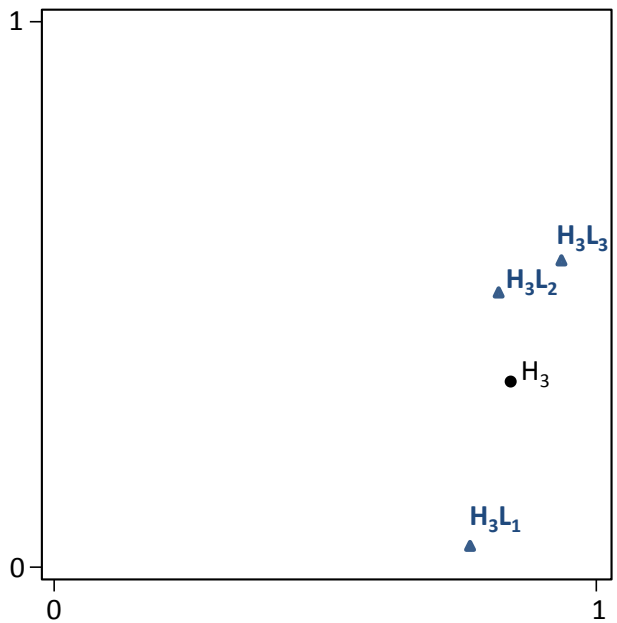
- L_q the level q of the hierarchy j

Level representation



Coordinate of level q on axis s:

$$\eta^2(z_s, L_q),$$

with:

- z_s the axis s
- L_q the level q

2 consequences:

- Levels ordered along each axis
- Hierarchy = barycenter of its levels

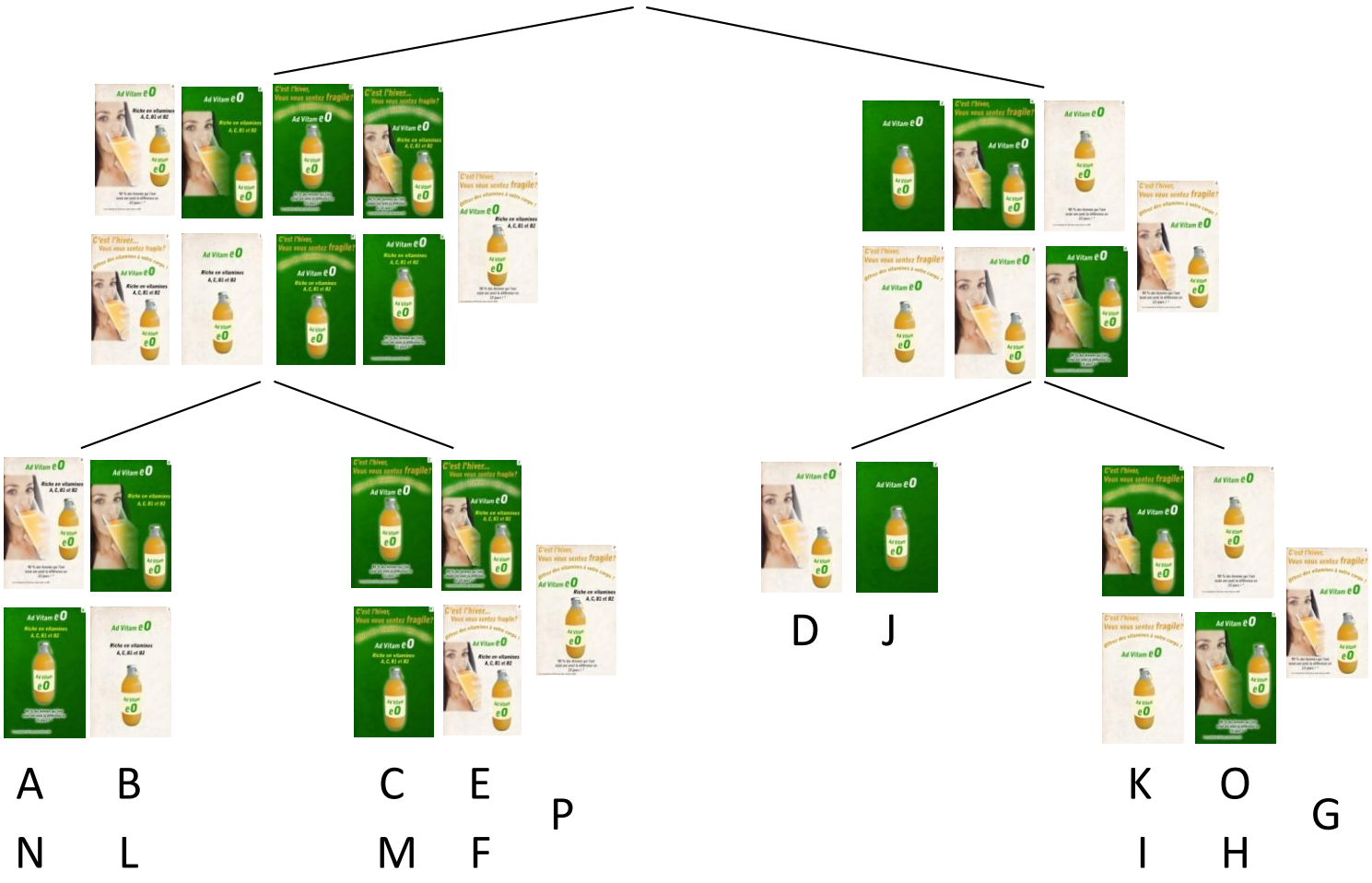
Outline

Data

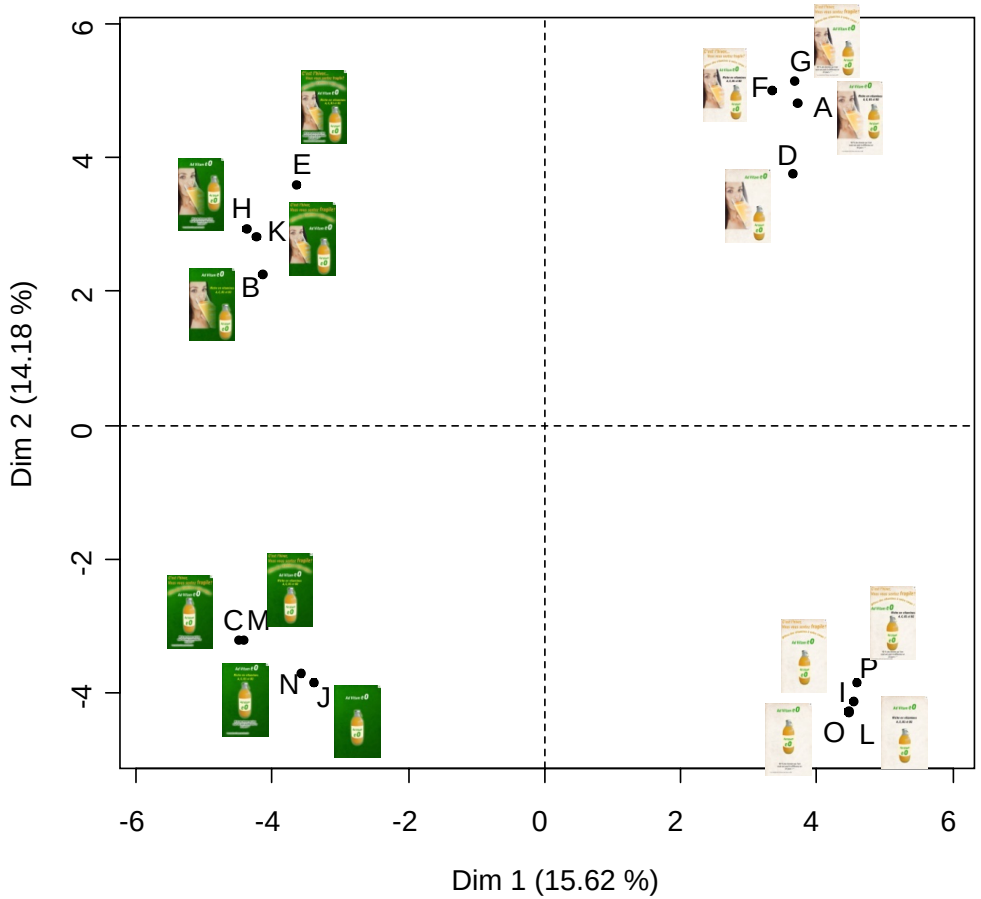
- 16 advertisements concerning an orange juice
- Advertisements built according to a 2^{5-1} fractional factorial design
- 22 subjects
- Hierarchical sorting

Example of hierarchical sorting: subject number 3

Example of hierarchical sorting: subject number 5

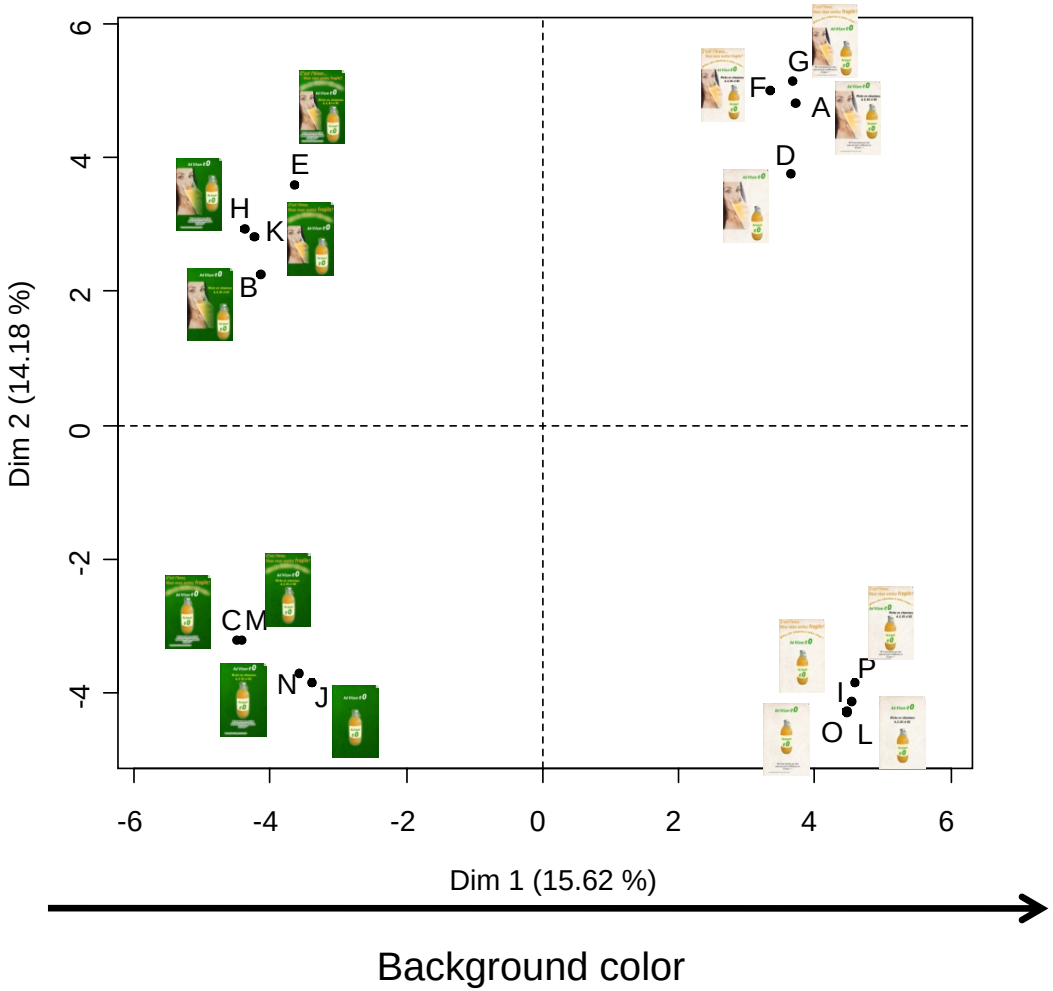


Advertisement representation



$$\lambda_1 = 16.55$$

Advertisement representation



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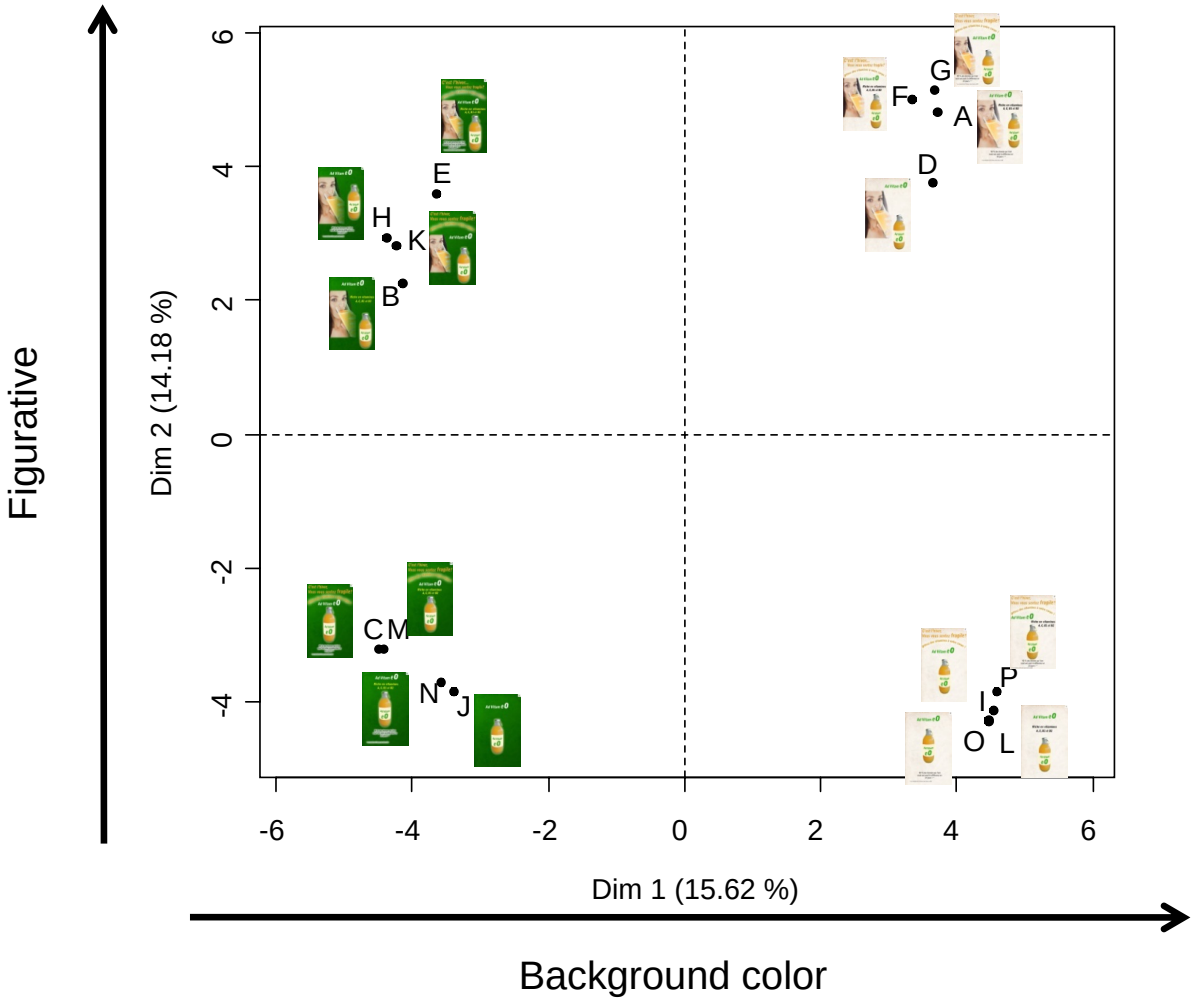
Advertisement representation



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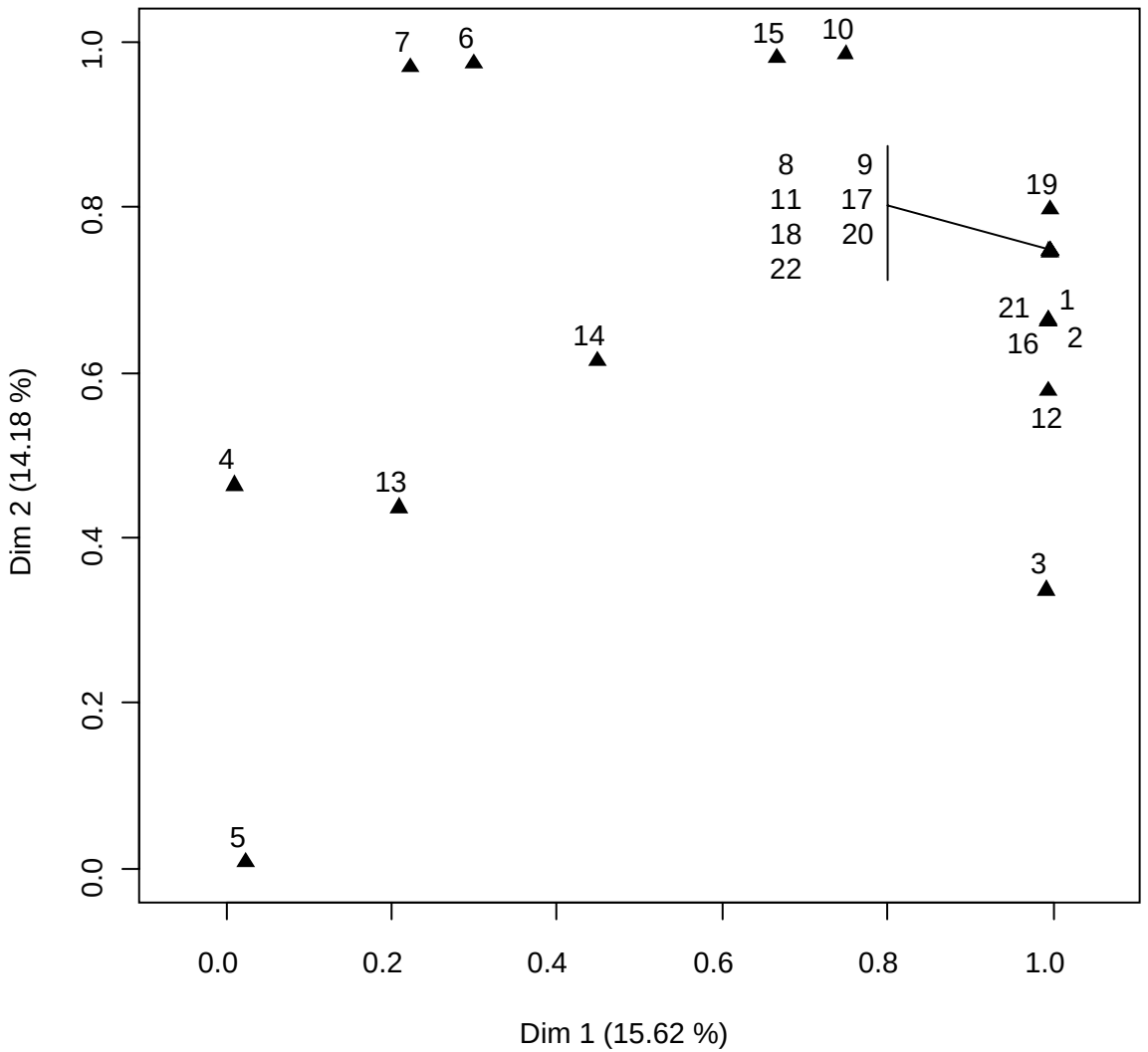
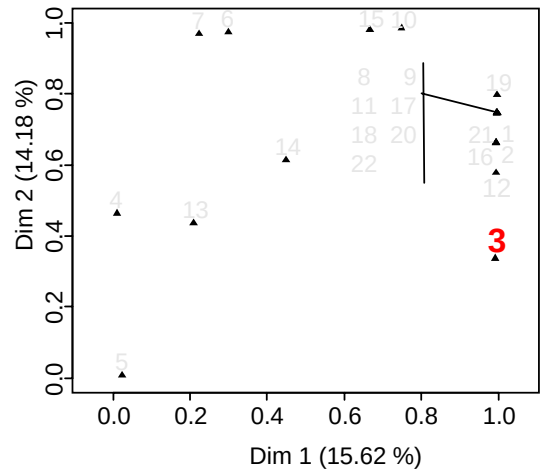
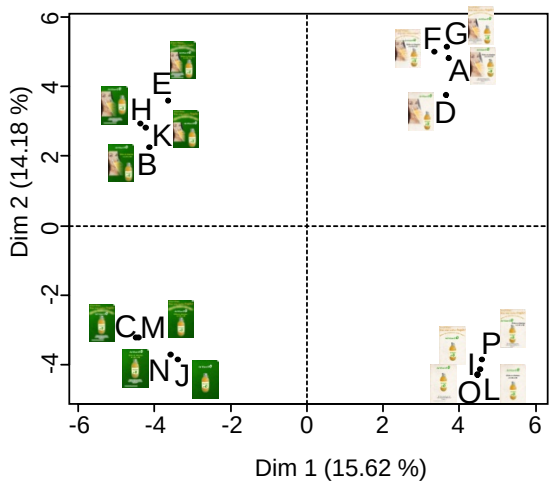
Hierarchy representation



Hierarchy representation: subject number 3





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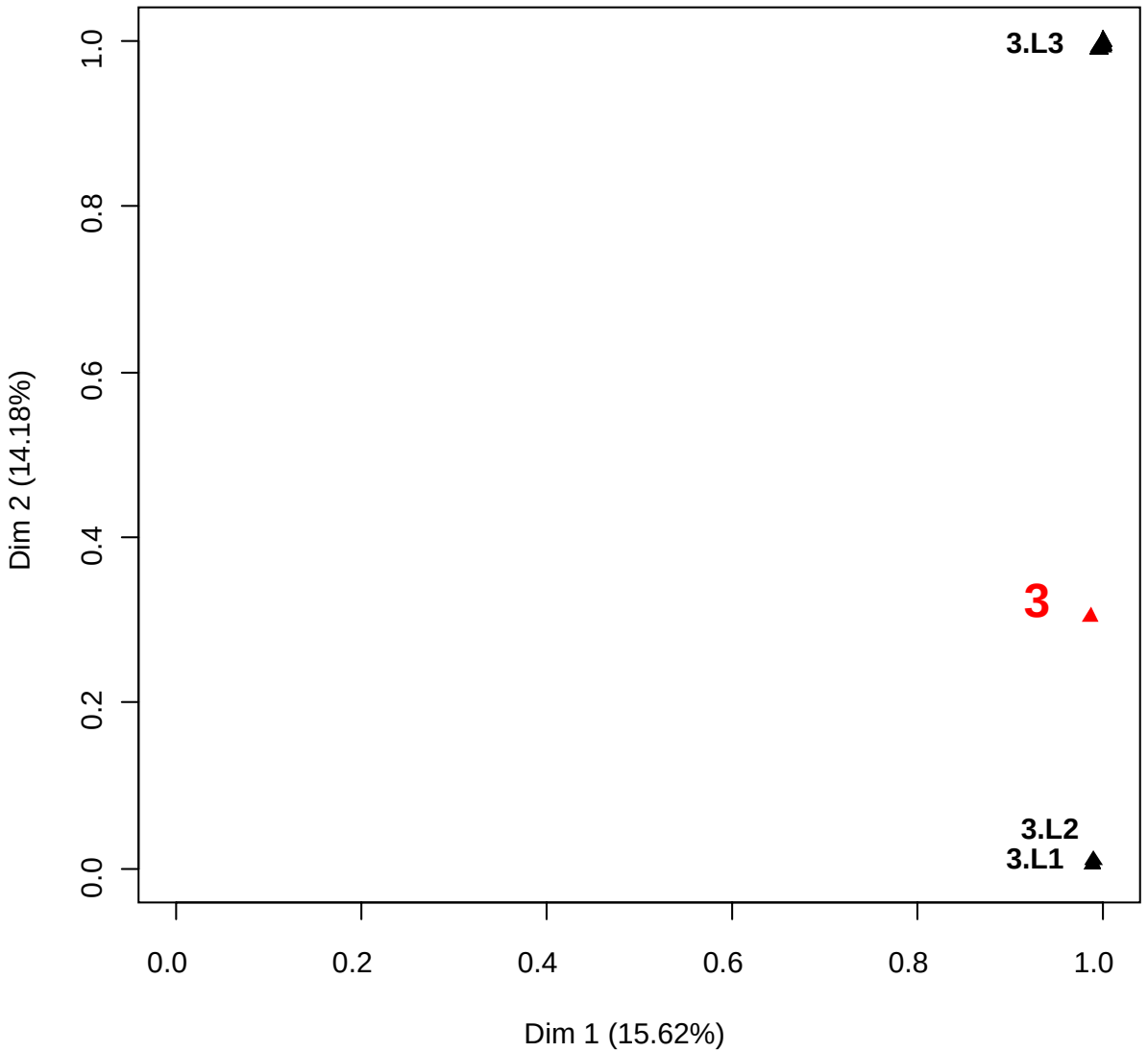
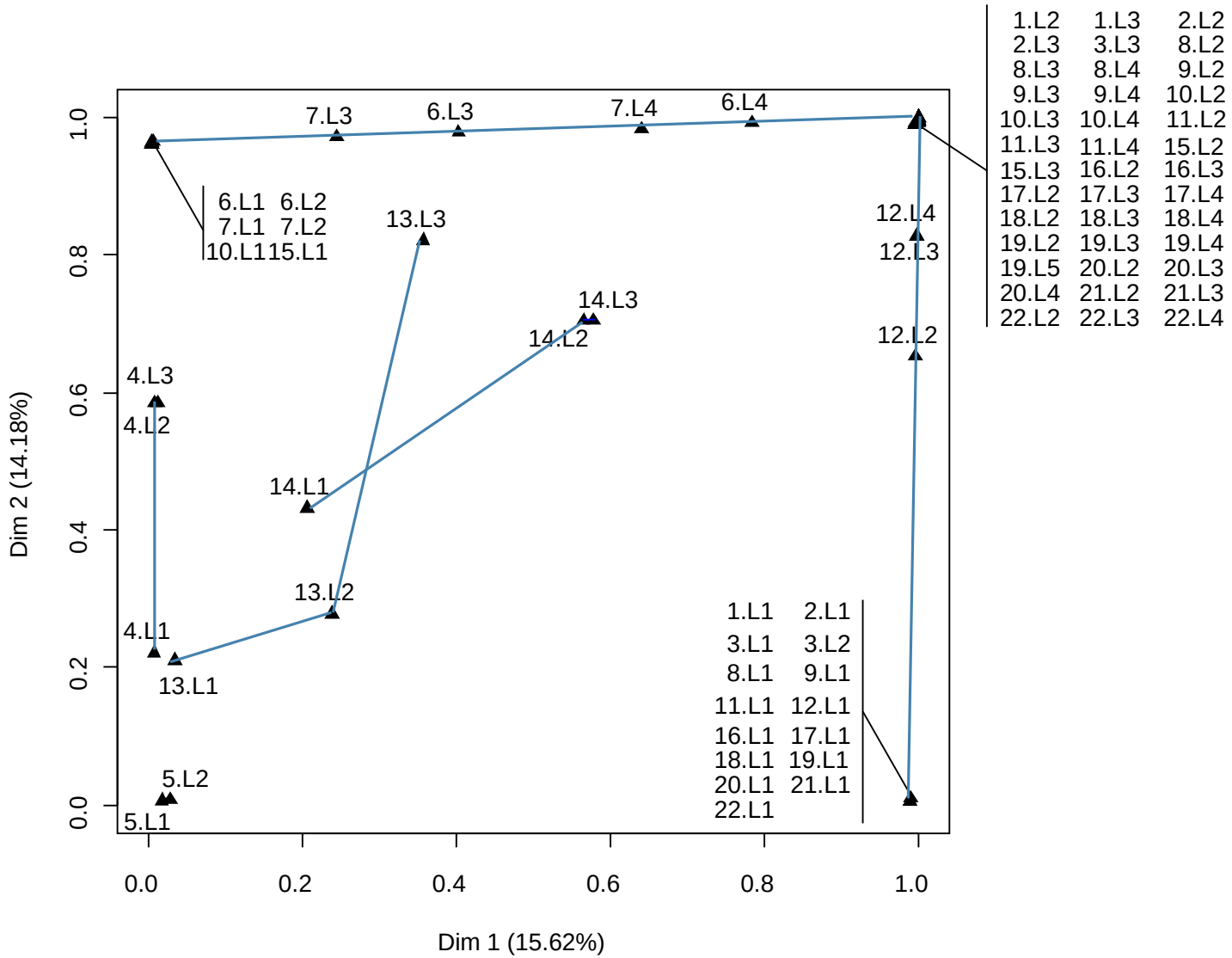


Level representation: trajectories



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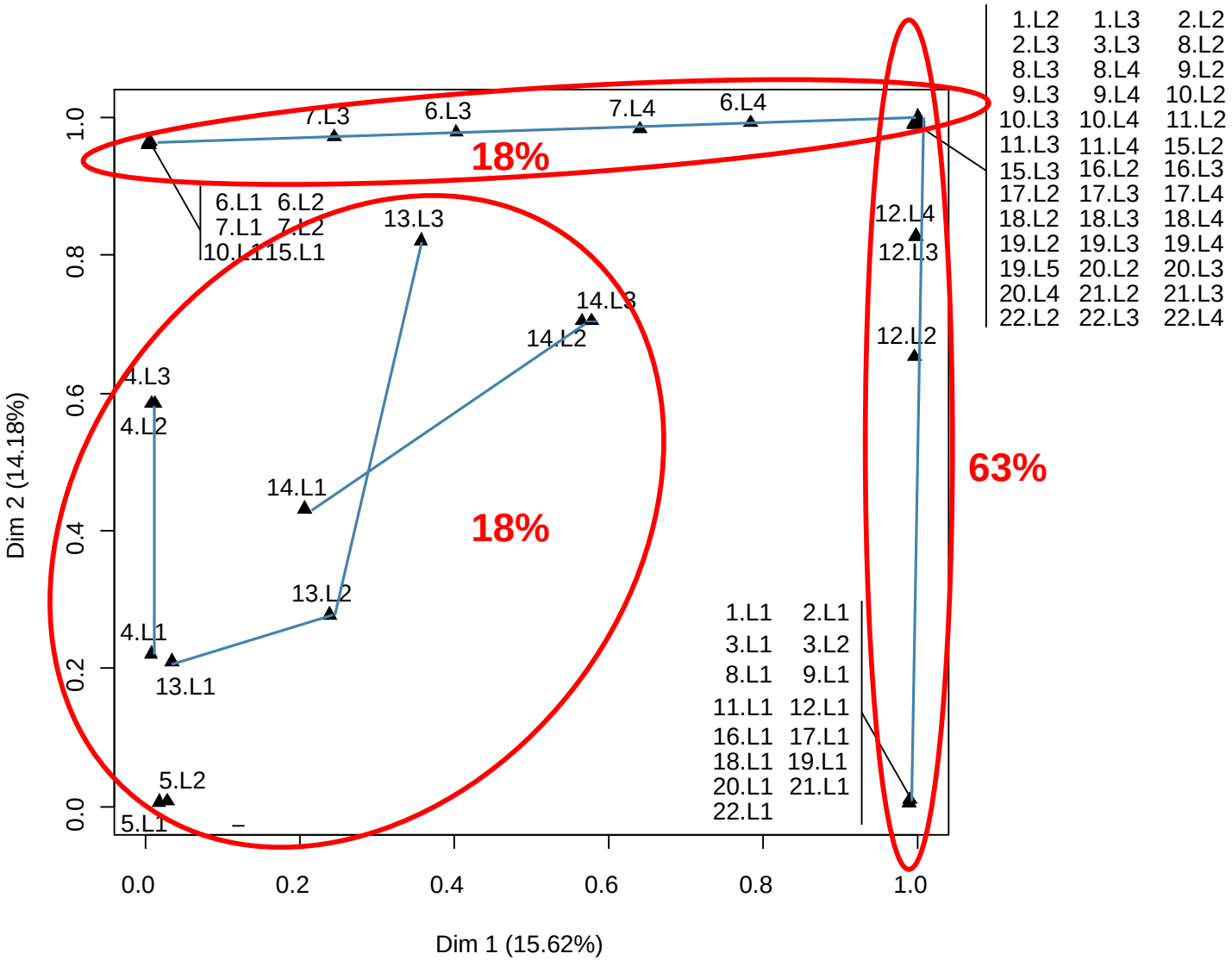

Outline

Conclusion

- Methodology providing:
  - Representation of objects, hierarchies, levels
  - Representations related to each other
  - Representations interpretable according to simple rules
- In the example, suggests groups of hierarchies
- Allows the simultaneous taking into account of hierarchies and partitions in a same analysis
- Program available in the SensoMineR package

References

Adams, E. I. (1972). Consensus techniques and the comparison of taxonomic trees. *Systematic Zoology*, 21:390–397.

Escofier, B. and Pagès, J. (1982). Comparaison de groupes de variables définies sur le même ensemble d'individus. *Rapport de recherche INRIA*, 149.